DAWSON COLLEGE
MATHEMATICS DEPARTMENT

FINAL EXAMINATION
Fall 2024

Remedial Activities for Secondary V Mathematics
201-015-RE

Instructor: R. Acteson & G. Chu

Student Name: ______________________________ Student ID: ________________

Date: December 19, 2024 Time: 9:30 – 12:30

Instructions:

- Print your name and student ID number in the space provided above.
- All questions are to be answered directly on the examination paper in the space provided.
- SHARP EL-531 calculators are permitted.

All questions are equally weighted.
This exam must be returned intact.

1. Perform the following long division: $\frac{2x^3-2x+4}{x-3}$

$$
\begin{array}{r}
2x^2 + 6x + 16 \\
x-3 \;\big)\; 2x^3 \qquad\quad - 2x + 4 \\
-\underline{(2x^3 - 6x^2)} \\
6x^2 - 2x + 4 \\
-\underline{(6x^2 - 18x)} \\
16x + 4 \\
-\underline{(16x - 48)} \\
52
\end{array}
$$

$$\boxed{2x^2+6x+16+\frac{52}{x-3}}$$

2. Simplify: $\frac{20}{x^2-2x-15} - \frac{24}{x^2-9}$

$(x-5)(x+3) \quad (x-3)(x+3)$

$$= \frac{20}{(x-5)(x+3)}\left(\frac{x-3}{x-3}\right) - \frac{24}{(x-3)(x+3)}\left(\frac{x-5}{x-5}\right)$$

$$= \frac{20x-60-24x+120}{(x-5)(x-3)(x+3)}$$

$$= \boxed{\frac{-4x+60}{(x-5)(x-3)(x+3)}}$$

3. Rationalize the denominator and simplify: $\frac{2}{\sqrt{20}-4}$ $\left(\frac{\sqrt{20}+4}{\sqrt{20}+4}\right)$

$$= \frac{2(\sqrt{4(5)}+4)}{20-16}$$

$$= \frac{2(2\sqrt{5}+4)}{4}$$

$$= \frac{\cancel{4}(\sqrt{5}+2)}{\cancel{4}}$$

$$\boxed{\sqrt{5}+2}$$

4. Solve the equation: $\sqrt{2x+24}-8=x$

$$\left(\sqrt{2x+24}\right)^2 = (x+8)^2$$

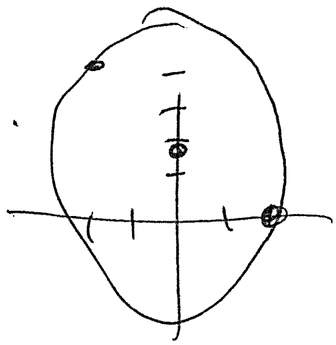
$$2x+24 = x^2+16x+64$$

$$0 = x^2+14x+40$$

$$0 = (x+10)(x+4)$$

$$\cancel{x=-10} \quad \text{or} \quad \boxed{x=-4}$$

5. Find an equation of the circle with (2, 0) and (-2, 4) as the 2 endpoints of the diameter.



Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2-2}{2}, \frac{0+4}{2}\right) = (0, 2) = \text{center}$

radius $= \sqrt{(0-2)^2+(2-0)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$ $\quad r^2 = 8$

Circle $(x-h)^2+(y-k)^2 = r^2$

$$\boxed{x^2+(y-2)^2 = 8}$$

6. Consider the points A(2, 0) and B(-2, 4). Find an equation of the line that passes through the midpoint of line segment AB and is also perpendicular to line segment AB.

Midpoint $= (0,2)$

slope $= \frac{\Delta y}{\Delta x} = \frac{4-0}{-2-2} = -1 \implies \perp$ slope $= 1$

$y = ax+b$

$y = 1x+b$ $\quad$ plug in $(0,2)$

$2 = 0+b$

$\boxed{b=2}$ $\quad \boxed{y = x+2}$

7. Find $\dfrac{f(x+h)-f(x)}{h}$ and simplify if $f(x) = x^2 + 4x$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+4(x+h)-(x^2+4x)}{h}$$

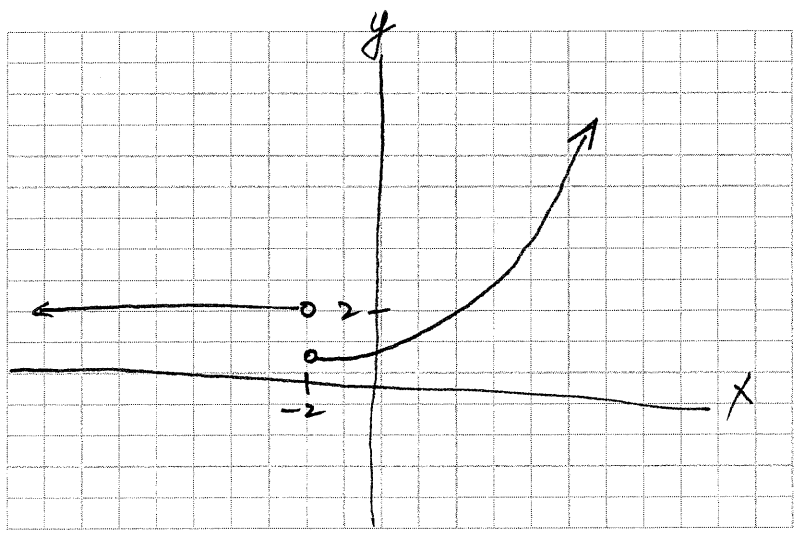
$$= \frac{\cancel{x^2}+2xh+h^2+\cancel{4x}+4h-\cancel{x^2}-\cancel{4x}}{h}$$

$$= \frac{2xh+h^2+4h}{h}$$

$$= \frac{\cancel{h}(2x+h+4)}{\cancel{h}} = \boxed{2x+h+4}$$

8. Graph the following piecewise defined function. Also, state the domain and the range.

$$f(x) = \begin{cases} 2, x < -2 \\ 2^x, x > -2 \end{cases}$$



Domain: $\mathbb{R} \setminus \{-2\}$

Range: $y > 1/4$

9. If $f(x) = \frac{x-24}{2}$ and $g(x) = log_2 x$, find $(f \circ g)(64)$ and $f^{-1}(x)$.

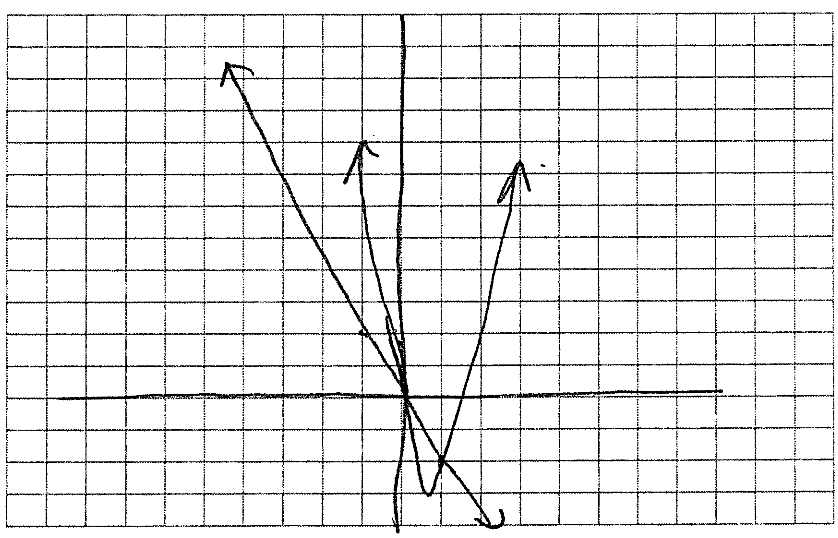
$$(f \circ g)(64) = f(g(64))$$
$$= f(\log_2(64))$$
$$= f(\log_2(2^6))$$
$$= f(6)$$
$$= \frac{6-24}{2} = -\frac{18}{2} = \boxed{-9}$$

$$y = f(x) = \frac{x-24}{2}$$
$$x = \frac{y-24}{2}$$
$$2x = y - 24$$
$$\boxed{y = 2x + 24 = f^{-1}(x)}$$

10. Find the intersection(s) of the following graphs and sketch them:

$$\begin{cases} y = -2x \\ y = 2x^2 - 4x \end{cases}$$

$$y = y$$
$$-2x = 2x^2 - 4x$$
$$0 = 2x^2 - 2x$$
$$0 = 2x(x-1)$$
$$\boxed{\begin{matrix} x=0 \\ y=0 \end{matrix}} \text{ or } \boxed{\begin{matrix} x=1 \\ y=-2 \end{matrix}}$$



11. Solve the equation: $5^{2x} = 3^{2+4x}$

$$\ln(5^{2x}) = \ln(3^{2+4x})$$

$$2x \ln 5 = (2+4x)\ln 3$$

$$2x \ln 5 = 2\ln 3 + 4x \ln 3$$

$$2x \ln 5 - 4x \ln 3 = 2\ln 3$$

$$\boxed{x = \frac{2\ln 3}{2\ln 5 - 4\ln 3} \approx -1.87}$$

12. Solve the equation: $log_{24}(x-2) + log_{24} x = 1$

$$\log_{24}((x-2)(x)) = \cancel{\log}_{24}(24)$$

$$x^2 - 2x = 24$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$$\underset{0}{\overset{\shortparallel}{}} \qquad \underset{0}{\overset{\shortparallel}{}}$$

$$\boxed{x = 6} \quad \text{or} \quad \cancel{x = -4}$$

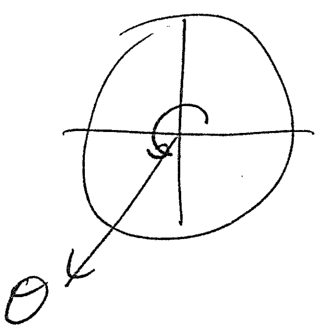
13. Express as a single log and simplify: $\ln(x^2+x-20)+ln(x^2+4x)-\ln(x^2-16)$

$$\ln\left(\frac{(x^2+x-20)(x^2+4x)}{x^2-16}\right) = \ln\left(\frac{(x+5)\cancel{(x-4)}\,x\,\cancel{(x+4)}}{\cancel{(x-4)}\cancel{(x+4)}}\right)$$

$$= \boxed{\ln(x^2+5x)}$$

14. Find the exact values of $\cos\theta$ and $\cos 2\theta$ if $cot\ \theta = \frac{20}{21}$ and θ is in quadrant III.

$$\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{20}{21}$$



$x = -20$

$y = -21$

$x^2+y^2=r^2$

$(-20)^2+(-21)^2=r^2$

$841 = r^2$

$r = \sqrt{841} = 29$

$$\boxed{\cos\theta = \frac{x}{r} = \frac{-20}{29}}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$= \left(\frac{-20}{29}\right)^2 - \left(\frac{-21}{29}\right)^2$$

$$= \boxed{\frac{-41}{841}}$$

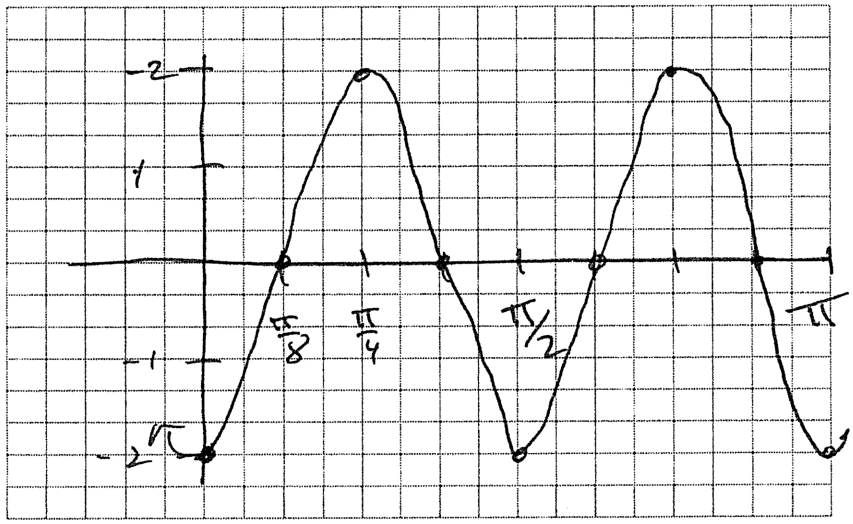
15. State the amplitude and the period of $y = -2\cos 4x$ and graph it for two cycles.

$\text{period} = \frac{2\pi}{b}$

$b = 4$

$\text{period} = \frac{2\pi}{4}$

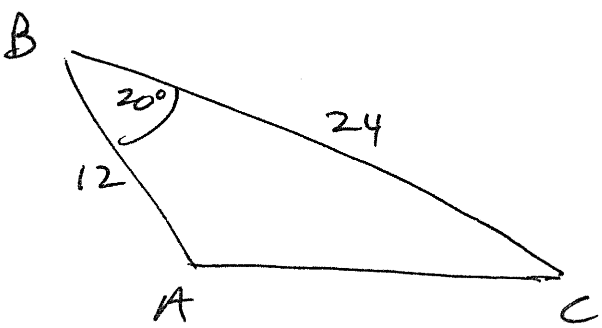
$= \frac{\pi}{2}$



Amplitude: 2

Period: $\pi/2$

16. Find b if $B = 20°$, $a = 24$ and $c = 12$ in the triangle ABC.



$b^2 = a^2 + c^2 - 2ac\cos B$

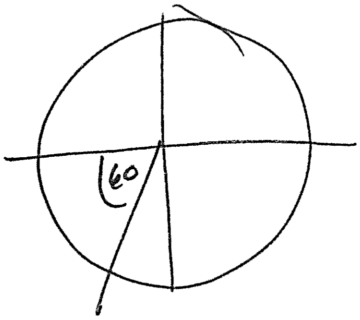
$= 24^2 + 12^2 - 2(24)(12)\cos(20°)$

$= 720 - 541.26$

$= 178.73$

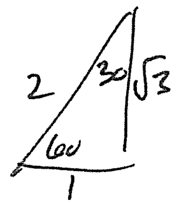
$b \approx \sqrt{178.73} \approx \boxed{13.36}$

17. Use the reference angle and the quadrant to find the exact value of $\csc(240°)$.



$\theta = 240°$

$\theta_{ref} = 60°$



$$\csc 240° = -\csc 60°$$
$$= \boxed{-\frac{2}{\sqrt{3}}}$$

Reference angle: $60°$

Quadrant: III

18. Verify the identity: $(2sinx - 2cosx)^2 + 4sin2x = 4$.

$$(2\sin x - 2\cos x)^2 + 4(2\sin x\cos x) = 4$$

$$4\sin^2 x - \cancel{8\sin x\cos x} + 4\cos^2 x + \cancel{8\sin x\cos x} = 4$$

$$4\sin^2 x + 4\cos^2 x = 4$$

$$4(\underbrace{\sin^2 x + \cos^2 x}_{=1}) = 4$$

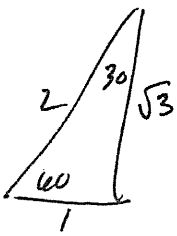
$$4(1) = 4 \checkmark$$

19. Solve the equation ($0 \le \theta < 360°$): $2sec\, x + 4 = 0$.

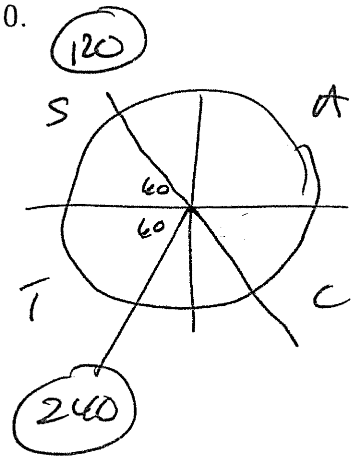
$2\sec x = -4$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

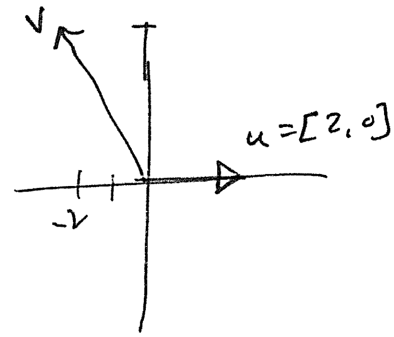


$\boxed{\cos 60° = \frac{1}{2}}$



$x = \underline{120° \text{ or } 240°}$

20. Find the angle in degrees between vectors $\vec{u} = [2, 0]$ and $\vec{v} = [-2, 4]$. Correct your answers to 4 decimal places.



$u \circ v = 2(-2) + 0(4)$

$= -4$

$u \circ v = \|u\| \, \|v\| \cos\theta$

$\|u\| = \sqrt{2^2 + 0^2} = 2$

$\cos\theta = \frac{u \circ v}{\|u\| \, \|v\|}$

$\|v\| = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$

$\cos\theta = \frac{-4}{2\sqrt{20}} \Rightarrow \theta = \cos^{-1}\left(\frac{-4}{2\sqrt{20}}\right) \approx \boxed{116.6°}$

Memory Aid for Remedial Activities for Secondary V Mathematics (201-015-RE)[1]

Trigonometric Functions

$$\sin\theta = y/r \quad \csc\theta = r/y$$
$$\cos\theta = x/r \quad \sec\theta = r/x$$
$$\tan\theta = y/x \quad \cot\theta = x/y$$

$$\sin\theta = \mathrm{O/H} \quad \csc\theta = \mathrm{H/O}$$
$$\cos\theta = \mathrm{A/H} \quad \sec\theta = \mathrm{H/A}$$
$$\tan\theta = \mathrm{O/A} \quad \cot\theta = \mathrm{A/O}$$

Trigonometric Identities

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(2x) = 2\sin x \cos x$$
$$\begin{aligned}\cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

Inverse Trigonometric Functions (range)

$\sin^{-1}$ or arcsin : $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $\qquad$ $\cos^{-1}$ or arccos : $0 \leq \theta \leq \pi$ $\qquad$ $\tan^{-1}$ or arctan : $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Trigonometric Laws

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 + b^2 - 2ab\cos C = c^2$$
$$b^2 + c^2 - 2bc\cos A = a^2$$
$$a^2 + c^2 - 2ac\cos B = b^2$$

Sectors of Circles

$$\theta = \frac{s}{r}, \qquad A = \frac{1}{2}\theta r^2$$

[1]Last updated: May 10, 2022