

DAWSON COLLEGE  
MATHEMATICS DEPARTMENT  
Remedial Activities Sec. V - MATHEMATICS

**I confirm that I have read and understood the College's Academic Integrity Document and will adhere to the principles of academic integrity while writing this exam.**

**201-015-RE S01-02**

**Winter 2023**

**Final Examination**

**May 19<sup>th</sup>, 2023**

**Time Limit: 3 hours**

Name: \_\_\_\_\_

ID#: \_\_\_\_\_

Instructors: A. Gambioli, I. Gorelyshev

- This exam contains 13 pages (including this cover page) and 17 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your solutions in a legible and orderly manner.
- You are only permitted to use the **Sharp EL-531\*\*** calculator.
- Good luck!

Question	Points	Score
1	5	
2	5	
3	6	
4	5	
5	10	
6	5	
7	4	
8	9	
9	6	
10	8	
11	5	
12	5	
13	6	
14	4	
15	5	
16	6	
17	6	
Total:	100	

1. (5 marks) Simplify, expressing your answer in terms of positive exponents only.

$$\frac{(3x^4y^{-2})^2(xy^2)^{-1}}{9^{-1}x^{-3}y}.$$

**Answer:**  $\frac{81x^{10}}{y^7}$

2. (5 marks) Divide by long division to find the quotient and the remainder:

$$\frac{2x^3 - 4x^2 + 2x + 1}{x + 1}$$

**Answer:**  $Q = 2x^2 - 6x + 8, R = -7$

3. (6 marks) Express as a single fraction in simplest form:

$$\frac{x^2 + 10x + 25}{2x^2 - x - 1} \div \frac{x^2 + 5x}{x^2 - 1}$$

**Answer:**  $\frac{(x+5)(x+1)}{x(2x+1)}$

4. (5 marks) Simplify completely:

$$2\sqrt{20} - 3\sqrt{18} + \sqrt{32} + 5\sqrt{45}$$

**Answer:**  $19\sqrt{5} - 5\sqrt{2}$

5. Solve for  $x$  and express your answer in simplest form:

(a) (5 marks)  $\frac{x}{x-1} - \frac{2}{x+1} = \frac{4x^2}{x^2-1}$

**Answer:**  $x = \frac{2}{3}$

(b) (5 marks)  $\sqrt{2x-1} + 2x - 1 = 0$

**Answer:**  $x = \frac{1}{2}$

6. (5 marks) Find  $y$  so that  $P(5, y)$  is equidistant from  $Q(1, 2)$  and  $R(7, -2)$

**Answer:**  $y = \frac{3}{2}$

7. (4 marks) Find the equation of the line that is parallel to the line passing through  $A(1, 2)$  and  $B(9, 6)$  and passes through  $C(2, 2)$ .

**Answer:**  $y = \frac{1}{2}x + 1$

8. Consider the function  $f(x) = x^2 - 4x + 3$ .

(a) (3 marks) Find the  $x$  and  $y$  intercepts of  $f(x)$ ;

**Answer:** y-int:  $(0, 3)$ ; x-int:  $((3, 0)$  and  $(1, 0)$ .

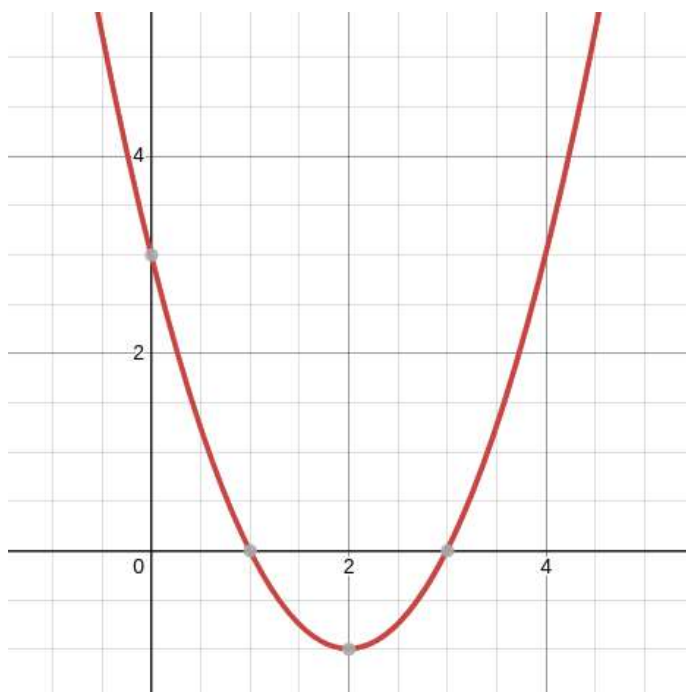
(b) (2 marks) find the  $x$  and  $y$  coordinates of the vertex of the graph of  $f(x)$ ;

**Answer:**  $v = (2, -1)$ .

(c) (2 marks) state the range and the axis of symmetry of  $f(x) = x^2 - 4x + 3$ ;

**Answer:**  $R = [-1, +\infty)$ , AOS:  $x = 2$ .

(d) (2 marks) plot the graph of  $f(x)$  on the grid below, showing clearly the intercepts, vertex and axis of symmetry.



9. State the domains of the given functions:

(a) (3 marks)  $f = \frac{2x-1}{x^3-25x}$

**Answer:**  $D = \mathbb{R} \setminus \{0.5, -5\}$ .

(b) (3 marks)  $g(x) = \sqrt{8+4x}$

**Answer:**  $D = [-2, +\infty)$ .



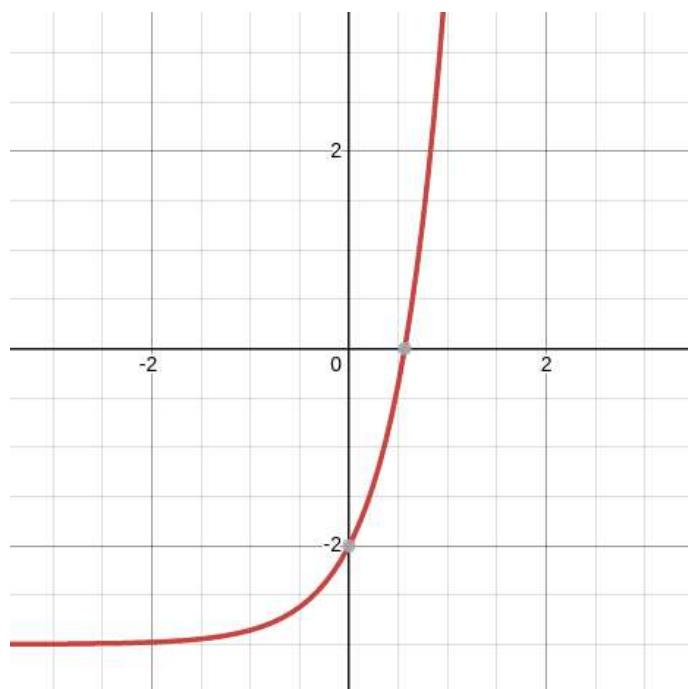
10. (a) (4 marks) Consider  $f(x) = 3x + 5$  and  $g(x) = 2x - 1$ . Find  $(f \circ g)(x)$  and simplify it.

**Answer:**  $(f \circ g)(x) = 6x + 2$ .

- (b) (4 marks) Consider the function  $f(x) = \frac{2x+1}{x-3}$ . Find and simplify  $f^{-1}(x)$ .

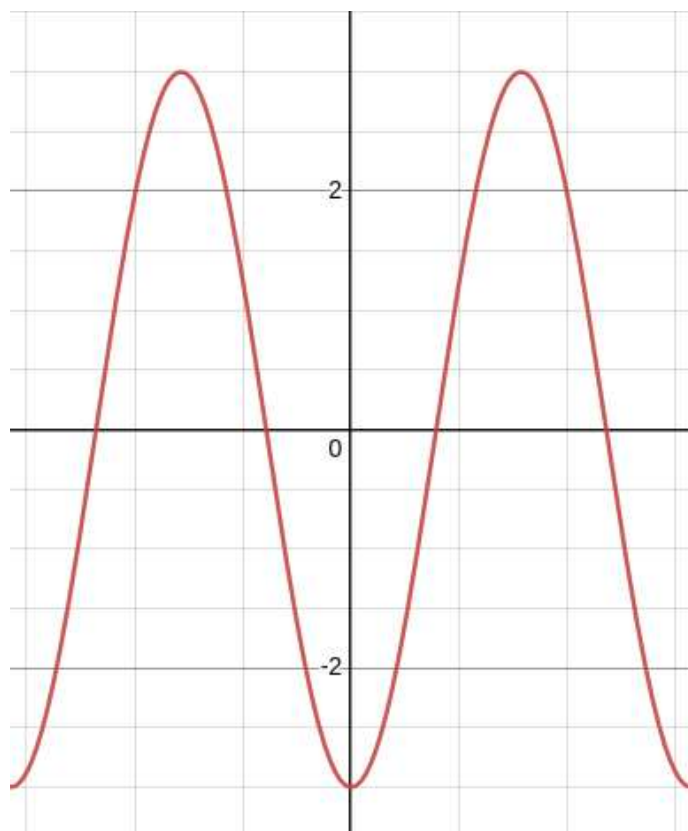
**Answer:**  $y = \frac{3x+1}{x-2}$ .

11. (5 marks) Sketch the graph of the function  $f(x) = 7^x - 3$  and state its asymptote, domain and range.



**Answer:**  $D = \mathbb{R}$ ,  $R = (-3, +\infty)$ , Hor. Asymp.:  $y = -3$ .

12. (5 marks) Sketch the graph of the function  $f(x) = -3\cos(4x)$  in the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .



13. (6 marks) Solve the triangle ABC such that  $a = 40$ ,  $b = 65$  and  $C = 50^\circ$ .

**Answer:**  $c = 49.82$ ,  $A = 37.94^\circ$ ,  $B = 92.05^\circ$ .

14. (4 marks) If the area of a disk sector is  $A = 25\pi \text{ cm}^2$  and the radius of the disk is  $r = 10\text{cm}$ , find the central angle  $\theta$  of the sector, both in radians and degrees.

**Answer:**  $\theta = \frac{\pi}{2} \text{ rad} = 90^\circ$ .

15. (5 marks) Consider the angle  $\theta = 120^\circ$ . Find the reference angle  $\theta_R$  and use it to find the exact value of  $\sec \theta$ .

**Answer:**  $\theta_R = 60^\circ$ ;  $\sec \theta = -2$ .

16. (6 marks) Verify the following trigonometric identity:

$$\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$$

**Answer:** Show that LS=RS, use the fact that  $\sec x = \frac{1}{\cos x}$  and  $\csc x = \frac{1}{\sin x}$ .

17. (6 marks) Find all the solutions of the following trigonometric equation in the interval  $[0^\circ, 360^\circ)$ :

$$2\sin^2(x)\cos(x) - \cos(x) = 0$$

**Answer:**  $x_1 = 90^\circ, x_2 = 270^\circ, x_3 = 45^\circ, x_4 = 135^\circ, x_5 = 225^\circ, x_6 = 315^\circ$ .