Dawson College
Department of Mathematics
Remedial Activities for Secondary V Mathematics
201-015-RE
Final Examination
Winter 2025

Student Name:———————————
Student I.D#:———————————
Teachers : M.Ishii, O.Zlotchevskaia
Date: May 21, 2025
Time: 9:30 am to 12:30 pm

Instructions:

- Print your name and ID in the provided space.
- This exam contains 11 pages (including this cover page) and 17 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Show all your steps and justify your answers.
- You are only permitted to use the **Sharp EL-531**** calculator.
- This examination booklet must be returned intact.
- Good luck!

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	4	
7	4	
8	5	
9	16	
10	8	
11	12	
12	5	
13	5	
14	4	
15	4	
16	4	
17	4	
Total	100	

1. (5 points) Simplify the expression and write the answer with positive exponents only.

$$\frac{(-3x^3y^2z)^2(-z^{-2})^3}{(x^{-3}y^4)^{-2}}$$

2. (5 points) Rationalize the denominator and simplify your answer $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}$.

3. (5 points) Perform the following long division: $\dfrac{2x^3 + 5x^2 + 9}{x + 3}$

4. (5 points) Simplify as much as possible $\dfrac{2x^2 + 3x + 1}{x^2 + 4x + 3} \div \dfrac{4x^2 - 1}{x^2 + 5x + 6}$

5. (5 points) Simplify as much as possible $\dfrac{\dfrac{1}{x+2} - \dfrac{1}{x+1}}{\dfrac{1}{x+1}}$

6. (4 points) The width of a rectangular lawn is 48 m less than the length. Find the dimensions of the lawn if its area is 2025 m^2;

7. (4 points) Solve for x $\quad \sqrt{3x+1} = 1 - \sqrt{x}$.

8. (5 points) Graph the following piecewise defined function. State the domain and the range.

$$f(x) = \begin{cases} x+1 & if \quad x < 1 \\ \log_2 x & if \quad x > 1 \end{cases}$$

9. Let $f(x) = \dfrac{2}{x+3}$, $\quad g(x) = 2x^2 + 5x$ and $\quad h(x) = \dfrac{x+2}{\sqrt{x-1}}$.

(a) (4 points) Calculate $5h(2) \cdot f(1) + 2g(-1)$

(b) (4 points) Find the domain of $h(x)$ .Write your answer using the solution graph and the interval notation.

(c) (4 points) Find $(g \circ f)(x)$ and simplify your answer as much as possible.

(d) (4 points) Find the inverse function of $f(x)$, that is $f^{-1}(x)$.

10. (a) (4 points) Find the intersection(s) of the graphs $y = x - 1$ and $y = x^2 - 4x + 3$.

(b) (4 points) Sketch the graphs of $y = x - 1$ and $y = x^2 - 4x + 3$. Find the vertex and the intercepts of the parabola and clearly show them on the graph.

11. Solve each equation

(a) (4 points) $8^{5x} = 5^{3x+1}$

(b) (4 points) $\log_4(x+2) + \log_4(x+8) = 2$

(c) (4 points) $2\ln x - \ln(x-4) = \ln(x+5)$

12. (5 points) Find the exact value of $sin(\theta)$ and $tan(\theta)$, if $cos(\theta) = \frac{3}{5}$ and $cot(\theta) < 0$.

13. (5 points) Solve triangle ABC given $\angle A = 28°, \angle B = 41°$ and $c = 15$. Approximate your answers to two decimal places.

14. (4 points) State the amplitude and the period of the function $y = -3sin(4x)$, then graph it for one period.

15. (4 points) Verify the identity: $cot\,(x) \cdot sin\,(2x) = 1 + cos\,(2x)$

16. (4 points) Solve the equation $2sin\,(x) - \sqrt{3} = 0$ in the interval $[0, 2\pi)$.

17. (4 points) Find the angle in degrees between vectors $\overrightarrow{u} = (1, -2)$ and $\overrightarrow{v} = (2, 3)$ Approximate your answer to two decimal places.

Memory Aid for Remedial Activities for Secondary V Mathematics (201-015-RE)[1]

Trigonometric Functions

$\sin\theta = y/r \quad \csc\theta = r/y$
$\cos\theta = x/r \quad \sec\theta = r/x$
$\tan\theta = y/x \quad \cot\theta = x/y$

$\sin\theta = \text{O/H} \quad \csc\theta = \text{H/O}$
$\cos\theta = \text{A/H} \quad \sec\theta = \text{H/A}$
$\tan\theta = \text{O/A} \quad \cot\theta = \text{A/O}$

Trigonometric Identities

$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\tan(-x) = -\tan x$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

Inverse Trigonometric Functions (range)

$\sin^{-1}$ or arcsin : $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $\quad \cos^{-1}$ or arccos : $0 \leq \theta \leq \pi$ $\quad \tan^{-1}$ or arctan : $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Trigonometric Laws

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 + b^2 - 2ab\cos C = c^2$
$b^2 + c^2 - 2bc\cos A = a^2$
$a^2 + c^2 - 2ac\cos B = b^2$

Sectors of Circles

$\theta = \frac{s}{r}, \quad A = \frac{1}{2}\theta r^2$

[1] Last updated: May 10, 2022

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1. $$\frac{(-3)^2 x^6 y^4 z^2 \cdot (-1)^3 z^{-6}}{x^8 y^{-8}} = -9 y^{12} z^{-4} = \boxed{-\frac{9y^{12}}{z^4}}$$

2. $$\frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{3^2-(\sqrt{3})^2} = \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6} = \boxed{2+\sqrt{3}}$$

3.
$$\begin{array}{r|l} & 2x^2 - x + 3 \\ \hline x+3 & 2x^3+5x^2+0x+9 \\ & 2x^3+6x^2 \\ \hline & -x^2+0x \\ & -x^2-3x \\ \hline & 3x+9 \\ & -3x+9 \\ \hline & 0 \end{array}$$

$\boxed{2x^2-x+3}$

4. $$\frac{2x^2+3x+1}{x^2+4x+3} \div \frac{4x^2-1}{x^2+5x+6} = \frac{(2x+1)(x+1)}{(x+1)(x+3)} \cdot \frac{(x+2)(x+3)}{(2x-1)(2x+1)}$$
$$= \boxed{\frac{x+2}{2x-1}}$$

5. $$\boxed{\frac{x+1-(x+2)}{(x+2)(x+1)} \cdot \frac{x+1}{1} = \frac{-1}{x+2}}$$

6. x is the length. $x(x-48) = 2025$

$x^2 - 48x - 2025 = 0$

$$x_{1,2} = \frac{48 \pm \sqrt{2304 - 4 \cdot 1 \cdot (-2025)}}{2}$$
$$= 24 \pm \frac{\sqrt{10404}}{2} = 24 \pm \frac{102}{2}$$
$$= 24 \pm 51 < \begin{matrix} 75 \\ -24 \end{matrix}$$

$x = 75$, $x - 48 = 27$

Ans $\boxed{75_m \times 27_m}$

7. $\sqrt{3x+1} = 1 - \sqrt{x}$

$3x+1 = 1 - 2\sqrt{x} + x$

$2x = -2\sqrt{x}$

$x = -\sqrt{x}$

$x^2 = x$

$x(x-1) = 0 \Rightarrow \boxed{x=0}$ $\quad x=1$

Verification:

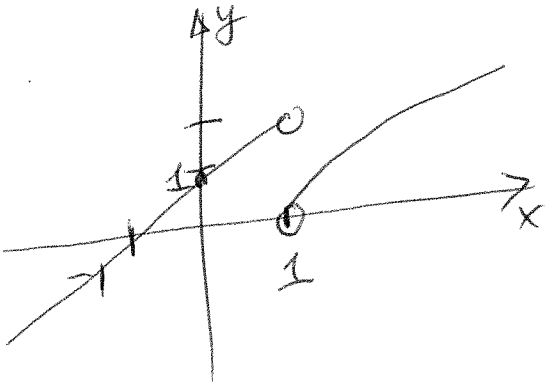
$x=1$ $\sqrt{3\cdot 1+1} \neq 1-\sqrt{1}$

rejected $2 \neq 0$

8) $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ \log_2 x & \text{if } x > 1 \end{cases}$

Domain $(-\infty, 1) \cup (1, \infty)$

Range $(-\infty, \infty)$



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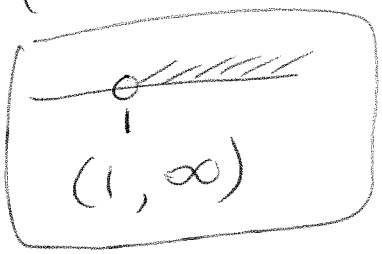
9. $f(x) = \frac{2}{x+3}$ $g(x) = 2x^2 + 5x$ $h(x) = \frac{x+2}{\sqrt{x-1}}$

a) $5h(2) \cdot f(1) + 2g(-1) = 5 \cdot \frac{2+2}{\sqrt{2-1}} \cdot \frac{2}{1+3} + 2 \cdot (2(-1)^2 + 5 \cdot (-1))$

$= 5 \cdot 4 \cdot \frac{2}{4} + 2(\overset{-3}{2-5}) = 10 - 6 = (4)$

b) Domain of $h(x)$: $x - 1 > 0$ $x > 1$



c) $(g \circ f)(x) = g(f(x)) = 2\left(\frac{2}{x+3}\right)^2 + 5\left(\frac{2}{x+3}\right)$

$= 2 \cdot \frac{4}{(x+3)^2} + \frac{10}{x+3} = \frac{8 + 10x + 30}{(x+3)^2} = \frac{10x + 38}{(x+3)^2}$

d) $f^{-1}(x)$: $y = \frac{2}{x+3}$ $x = \frac{2}{y+3}$ $y + 3 = \frac{2}{x}$ $y = \frac{2}{x} - 3 = \text{or } \frac{2-3x}{x}$

$f'(x) =$

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(10) $x^2 - 4x + 3 = x - 1$

a) $x^2 - 5x + 4 = 0$

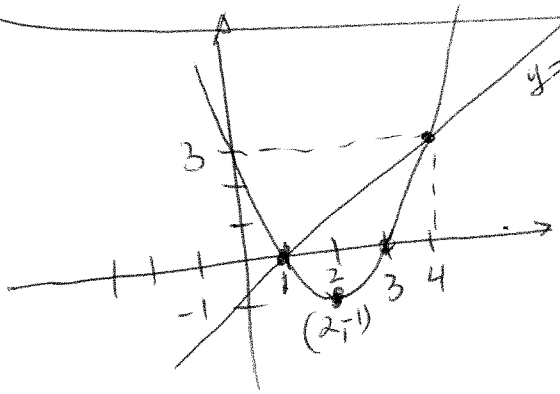
$(x-4)(x-1) = 0$

$x = 4, \quad x = 1$

$y = 3, \quad y = 0$

$(4, 3), (1, 0)$

b)



$x_{\text{vertex}} = -\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$

$y = 2^2 - 4 \cdot 2 + 3 = 4 - 8 + 3 = -1$

Vertex $(2, -1)$

$x^2 - 4x + 3 = 0$ $x = 3$

$(x-3)(x-1) = 0$ $x = 1$

$(3, 0)$ $(1, 0)$

$(0, 3)$

11) a) $8^{5x} = 5^{3x+1}$

$5x \log 8 = (3x+1)\log 5$

$5x\log 8 - 3x\log 5 = \log 5$

$x(5\log 8 - 3\log 5) = \log 5$

$$\boxed{x = \frac{\log 5}{5\log 8 - 3\log 5}}$$

b) $\log_4 (x+2)(x+8) = 2$

$4^2 = x^2 + 10x + 16$

$0 = x^2 + 10x$

$0 = x(x+10)$

$\boxed{x=0}$ $x = -10$ rejected.

Ans.

c) $\ln\left(\frac{x^2}{x-4}\right) = \ln(x+5)$

$x^2 = (x-4)(x+5)$

$x^2 = x^2 - 4x + 5x - 20$

$0 = x - 20$

$\boxed{x = 20}$

12) $\cos\theta = \frac{3}{5}$ $\cot\theta < 0$ $\tan\theta < 0$

Q4: $\cos\theta = \frac{x}{r}$, $x=3$, $r=5$

$y = \pm\sqrt{5^2 - 3^2} = \pm 4$

$\boxed{y = -4}$

$\sin\theta = \frac{y}{r} = -\frac{4}{5}$

$\tan\theta = -\frac{y}{x} = -\frac{4}{3}$

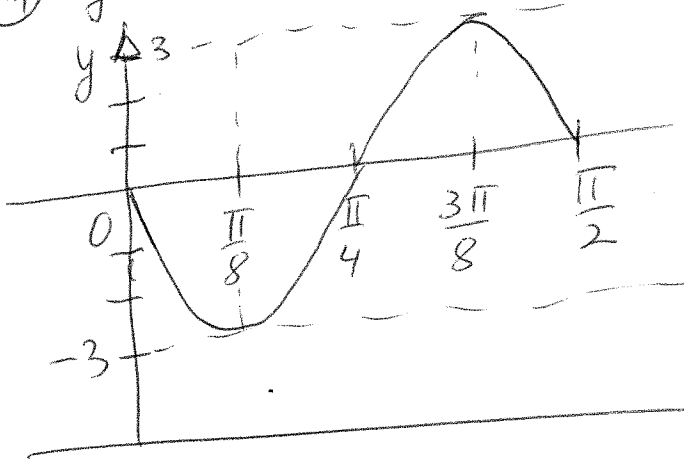
13)

$\angle C = 180° - 28° - 41° = 111°$

$\frac{a}{\sin(28)} = \frac{15}{\sin(111°)} \Rightarrow \boxed{a \approx 7.54}$

$\frac{b}{\sin 41°} = \frac{15}{\sin(111°)} \Rightarrow \boxed{b \approx 10.54}$

14) $y = -3\sin 4x$ $P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$



$\cot(x)\cdot\sin(2x) = 1 + \cos(2x)$

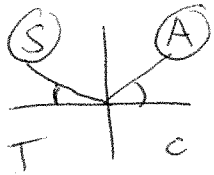
$\frac{\cos x}{\sin x}\cdot 2\sin x\cdot\cos x = 1 + 2\cos^2 x - 1$

$2\cos^2 x = 2\cos^2 x$

) $2\sin x - \sqrt{3} = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$x_R = \frac{\pi}{3}$



x_1: $x = x_R = \boxed{\frac{\pi}{3}}$

x_2: $x = \pi - x_R = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

17) $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|} = \frac{1\cdot 2 - 2\cdot 3}{\sqrt{1^2+(-2)^2}\sqrt{2^2+3^2}} = \frac{-4}{\sqrt{5}\sqrt{13}}$

$\approx \boxed{119.74°}$