DAWSON COLLEGE

MATHEMATICS DEPARTMENT

Rem. Act. for Sec. V Mathematics

201-015-RE 1, 2, 3
Fall 2019
Final Exam
December 13, 2019
Time Limit: 3 hours

Name: Solutions

ID#: ____________________

- This test contains 12 pages (including this cover page) and 18 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner.
- You are only permitted to use the **Sharp EL-531XG** or **Sharp EL-531X** calculator.
- This examination booklet must be returned intact.
- Good luck!

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	12	
8	4	
9	4	
10	8	
11	20	
12	4	
13	4	
14	4	
15	4	
16	4	
17	4	
18	4	
Total:	100	

1. (4 points) Simplify and express using only positive exponents.

$$\left(\frac{a^{-2}b^3}{-2a^3b^{-2}}\right)^{-3}.$$

$$= \frac{a^6 b^{-9}}{(-2)^{-3} a^{-9} b^6} = \frac{(-2)^3 a^6 a^9}{b^6 b^9} = \frac{-8a^{15}}{b^{15}}$$

2. (4 points) Find the quotient and remainder by long division of the expression

$$\frac{2x^3 - x^2 + 5x + 7}{2x + 1}$$

$$\begin{array}{r} x^2 - x + 3 \\ 2x+1 \overline{)\, 2x^3 - x^2 + 5x + 7} \\ -(2x^3 + x^2) \\ \hline -2x^2 + 5x + 7 \\ -(-2x^2 - x) \\ \hline 6x + 7 \\ -(6x + 3) \\ \hline 4 \end{array}$$

$\therefore Q = x^2 - x + 3$

$R = 4$

3. (4 points) Express as a single reduced fraction.

$$\frac{x^2-3x}{x^2+2x-15} \div \frac{x^2-3x-10}{x^2-25}$$

$$\frac{x\cancel{(x-3)}}{\cancel{(x+5)}\cancel{(x-3)}} \cdot \frac{\cancel{(x+5)}\cancel{(x-5)}}{\cancel{(x-5)}(x+2)}$$

$$= \frac{x}{x+2}$$

4. (4 points) Rationalize the denominator and simplify your answer.

$$\frac{\sqrt{3}}{2-\sqrt{3}} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{2\sqrt{3}+3}{4-3} = \frac{2\sqrt{3}+3}{1}$$

$$= 2\sqrt{3}+3$$

5. (4 points) Solve the inequality, giving the answer both in interval form and as a graph.

$$\left(2 < \frac{14-3x}{4} < 5\right) \cdot 4$$

$$8 < 14 - 3x < 20$$

$$8 - 14 < -3x < 20 - 14$$

$$\frac{-6}{-3} < \frac{-3x}{-3} < \frac{6}{-3}$$

$$2 > x > -2$$

$\Rightarrow x \in (-2, 2)$

6. (4 points) Sketch the functions given by $y = x^2 - 2x$ and $y = 6 - x$, and find their points of intersection.

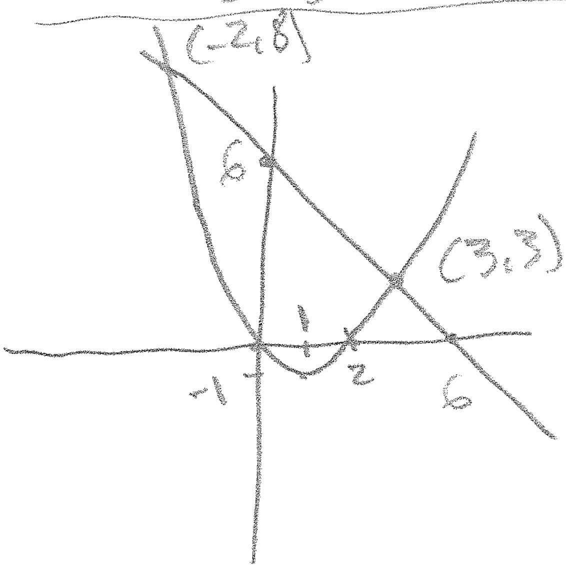
$y = x^2 - 2x = x(x-2) = 0 \Rightarrow x = 0, x = 2$

$x_v = \frac{-(-2)}{2(1)} = 1$, $y_v = 1^2 - 2(1) = -1$

$y = 6 - x$

$\Rightarrow$ slope $= -1$

y-int $= 6$



$x^2 - 2x = 6 - x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \Rightarrow y = 6 - 3 = 3$

$x = -2 \Rightarrow y = 6 + 2 = 8$

$\therefore$ (3,3) and (-2,8)

7. Solve and give the exact solutions in simplified form.

(a) (4 points) $x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(2)}}{2} = \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

(b) (4 points) $9^{2x+3} = 27^{4-4x}$

$$(3^2)^{2x+3} = (3^3)^{4-4x}$$

$$2(2x+3) = 3(4-4x)$$

$$4x + 6 = 12 - 12x$$

$$16x = 6$$

$$x = \frac{6}{16} = \frac{3}{8}$$

(c) (4 points) $\log_2(12x - 5) - 3 = \log_2(2x - 1)$

$$\log_2(12x-5) - \log_2(2x-1) = 3$$

$$\log_2\left(\frac{12x-5}{2x-1}\right) = 3$$

$$\frac{12x-5}{2x-1} = 2^3$$

$$12x - 5 = 8(2x-1)$$

$$12x - 5 = 16x - 8$$

$$8 - 5 = 16x - 12x$$

$$3 = 4x$$

$$x = 3/4$$

8. (4 points) Given that $f(x) = 2x^2 + 4x$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{(2(x+h)^2 + 4(x+h)) - (2x^2 + 4x)}{h}$$

$$= \frac{2(\cancel{x^2} + 2xh + h^2) + \cancel{4x} + 4h - \cancel{2x^2} - \cancel{4x}}{h}$$

$$= \frac{4x\cancel{h} + 2h^{\cancel{2}} + 4\cancel{h}}{\cancel{h}} = 4x + 2h + 4$$

9. (4 points) Given $f(x) = \dfrac{x+1}{3x-4}$, find $f^{-1}(x)$.

$$x = \frac{y+1}{3y-4}$$

$$3xy - 4x = y + 1$$

$$3xy - y = 1 + 4x$$

$$y(3x-1) = 1 + 4x$$

$$\therefore f^{-1}(x) = \frac{1+4x}{3x-1}$$

10. Let $f(x) = \sqrt{x^2 - 9}$ and $g(x) = 2x^2 + 1$.

(a) (4 points) Find $2f(5) - g(3)$.

$$= 2\sqrt{25-9} - (2(3)^2+1)$$

$$= 2\sqrt{16} - (19)$$

$$= 8 - 19$$

$$= -11$$

(b) (4 points) Find and simplify both $(f \circ g)(x)$ and $(g \circ f)(x)$

$$f(g(x)) = f(2x^2+1) = \sqrt{(2x^2+1)^2 - 9}$$

$$= \sqrt{4x^4 + 4x^2 + 1 - 9}$$

$$= \sqrt{4x^4 + 4x^2 - 8}$$

$$= 2\sqrt{x^4 + x^2 - 2}$$

$$g(f(x)) = g(\sqrt{x^2-9}) = 2(\sqrt{x^2-9})^2 + 1$$

$$= 2(x^2-9) + 1$$

$$= 2x^2 - 18 + 1 = 2x^2 - 17$$

11. Consider the points $A = (2, 4)$ and $B = (10, 20)$.

(a) (4 points) Find the slope of the line through A and B.

$$m = \frac{20-4}{10-2} = \frac{16}{8} = 2$$

(b) (4 points) Find the equation of the line through A and B.

$$y - 4 = 2(x-2)$$

$$y = 2x - 4 + 4$$

$$y = 2x$$

$A = (2, 4) \qquad B = (10, 20)$

(c) (4 points) Find the distance from A and B.

$$d = \sqrt{(2-10)^2 + (4-20)^2}$$
$$= \sqrt{64 + 256}$$
$$= \sqrt{320} = 8\sqrt{5}$$

(d) (4 points) Find the midpoint of A and B.

$$M = \left(\frac{2+10}{2}, \frac{4+20}{2}\right) = (6, 12)$$

(e) (4 points) Find the equation of the circle having A and B as endpoints of a diameter.

centre : $(6, 12)$, radius $= \frac{8\sqrt{5}}{2} = 4\sqrt{5}$

$$(x-6)^2 + (y-12)^2 = (4\sqrt{5})^2$$
$$(x-6)^2 + (y-12)^2 = 80$$

12. (4 points) Find the equation for the line passing through $(1, 2)$ and perpendicular to the line $y = \frac{1}{3}x + 8$.

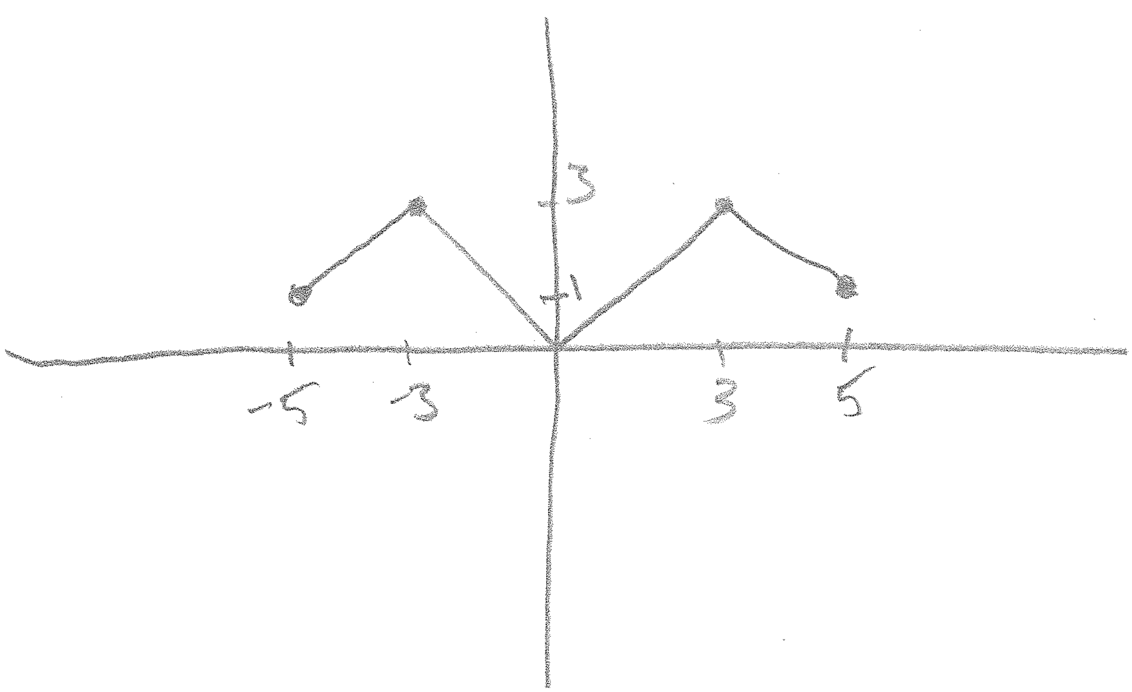
$m = \frac{-1}{1/3} = -3$

$y - 2 = -3(x - 1)$

$y = -3x + 3 + 2$

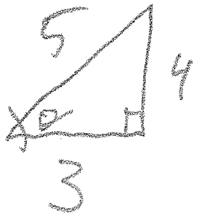
$y = -3x + 5$

13. (4 points) Sketch the graph of $f(x) = \begin{cases} 6 + x & \text{, if } -5 \le x < -3 \\ |x| & \text{, if } -3 \le x \le 3 \\ 6 - x & \text{, if } 3 < x \le 5 \end{cases}$



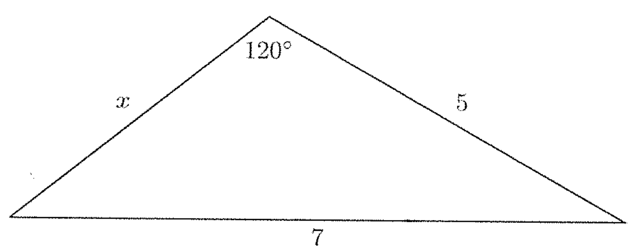
14. (4 points) Given that $\sin\theta = -\frac{4}{5}$ and $\tan\theta > 0$, find $\cos\theta$.

$\sin\theta < 0$ and $\tan\theta > 0 \Rightarrow$ QIII



$\Rightarrow \cos\theta = -3/5$

15. (4 points) Find the exact value of x:



$7^2 = x^2 + 5^2 - 2(x)(5)\cdot\cos(120°)$

$49 = x^2 + 25 - 10x(1/2)$

$0 = x^2 - 5x - 24$

$0 = (x-8)(x+3) \Rightarrow x = 8$ or $\cancel{x = -3}$

16. (4 points) Prove the identity $\cos(2\theta) + \sin(2\theta)\tan\theta = 1$

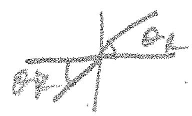
$$\left(1 - 2\sin^2\theta\right) + \left(2\sin\theta\cancel{\cos\theta}\right)\cdot\frac{\sin\theta}{\cancel{\cos\theta}}$$

$$= 1 - \cancel{2\sin^2\theta} + \cancel{2\sin^2\theta}$$

$$= 1$$

17. (4 points) Solve the equation $3\tan\theta - \sqrt{3} = 0$ in the interval $[0, 2\pi)$.

$3\tan\theta = \sqrt{3}$



$\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore\ \theta_1 = \pi/6$

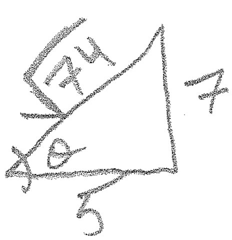
$\Rightarrow \theta_R = \pi/6$

$\theta_2 = \pi + \pi/6 = 7\pi/6$

18. (4 points) Find the exact value of $\cos\left(\arctan\left(-\frac{7}{5}\right)\right)$.

$\theta = \arctan\left(-\frac{7}{5}\right) \Rightarrow \theta$ is in QIV

$\Rightarrow \cos\theta > 0.$



$\Rightarrow \cos\left(\arctan\left(-\frac{7}{5}\right)\right) = \frac{5}{\sqrt{74}}$