

FINAL EXAMINATION (201-015-RE - S02)
Dawson College, Mathematics Department
Remedial Activities for Secondary V Mathematics (201-015-RE)

Winter 2022 Semester

May 20th, 2022

Time Limit: 3 hours

Instructors: G. Chu, E. Richer

Name: _____

ID#: _____

- This exam contains 14 pages (including this cover page) and 19 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner.
- You are only permitted to use the **Sharp EL-531X, XG or XT** calculator.
- This examination booklet must be returned intact.
- Good luck!

Question	Points	Score
1	6	
2	4	
3	5	
4	5	
5	4	
6	5	
7	4	
8	4	
9	6	
10	10	
11	4	
12	4	
13	5	
14	5	
15	5	
16	5	
17	6	
18	8	
19	5	
Total:	100	

1. (6 marks) (a) Rationalize the denominator and simplify:

$$\frac{7 + \sqrt{3}}{2 - \sqrt{3}}$$

- (b) Simplify the expression as much as possible. Your answer should be expressed as an **exact value** (i.e. no decimals).

$$\sqrt{24} - \sqrt{150} - \sqrt{54}$$

2. (4 marks) Solve for x .

$$8 - 5\left(\frac{3}{5}x + 2\right) = 2(3 - x) - 1$$

3. (5 marks) Solve for x .

$$1 - \frac{12}{x^2 - 4} = \frac{3}{x + 2}$$

4. (5 marks) Solve for x .

$$\sqrt{x} + \sqrt{x + 5} = 5$$

5. (4 marks) Find the **exact value** of $\sin \theta$ given $\sec \theta = -\frac{13}{5}$ and $\tan \theta < 0$

6. (5 marks) Show how you would use special angles to find the **exact value** of

$$\cos 300^\circ + \tan 135^\circ$$

7. (4 marks) Solve for x .

$$(2x + 3)(2x^2 - 5x - 3) = 0$$

8. (4 marks) Solve the system of equations.

$$\begin{cases} 5x + 2y = 1 \\ 2x + 3y = -15 \end{cases}$$

9. (a) (4 marks) Find the inverse of

$$f(x) = \frac{x-3}{2x-1}$$

- (b) (2 marks) **Explain** how you would show that the function you found in part (a) was in fact the inverse of $f(x)$ (you do not have to do any computations, simply state what you would have to do).

10. (10 marks) Given the following functions:

$$f(x) = 3x^2 + 1$$

$$g(x) = \sqrt{x - 3}$$

$$k(x) = \frac{x - 1}{2x + 1}$$

$$j(x) = 5 - 3x$$

Find the following:

(a) The domains of the $k(x)$ and $g(x)$

(b) $g(3)$

(c) $(g \circ j)\left(-\frac{8}{3}\right)$

(d) $(k \circ f)(x)$

(e) $(f \circ j)(x)$

(f) $\frac{f(x + h) - f(x)}{h}$

11. (4 marks) Find the **exact value** of $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

12. (4 marks) Solve triangle ABC; given $a = 20$, $C = 33^\circ$, and $b = 10$. Please round your answer to 3 decimal places.

13. (5 marks) Solve for x .

$$2^{1-3x} = 3 \cdot 5^{2x+1}$$

14. (5 marks) Solve for x .

$$\log_6(x+3) = 1 - \log_6(x+4)$$

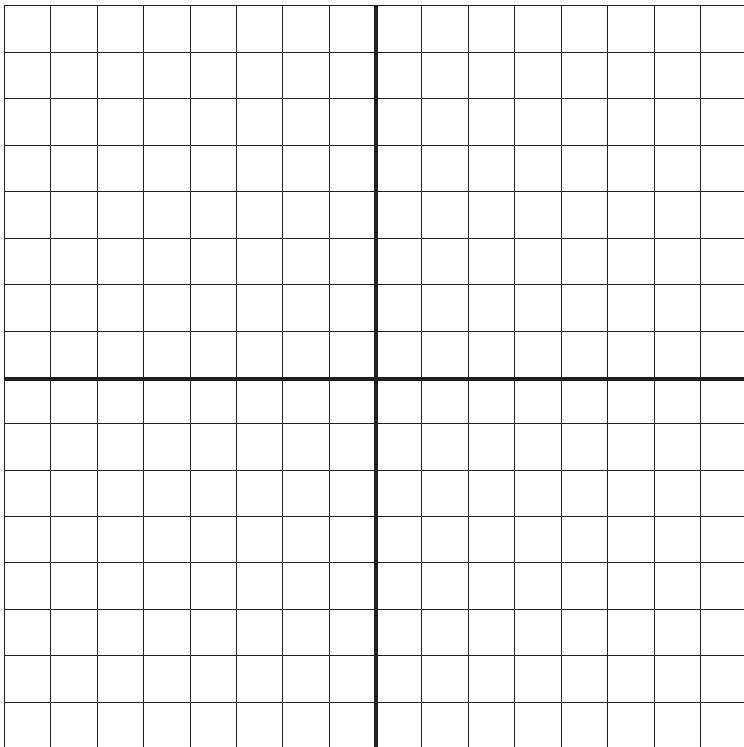
15. (5 marks) Consider the function

$$f(x) = \log_2(x - 1)$$

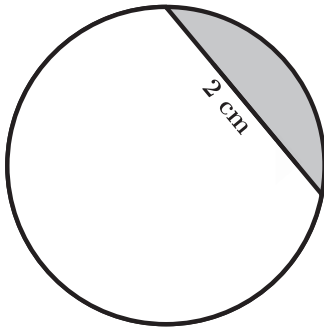
(a) State the domain $f(x)$

(b) State the range of $f(x)$

(c) Plot $y = f(x)$ in the space provided below. Include at least 4 different properly labelled points. Make sure that the general shape of the function is clearly illustrated.



16. (5 marks) Find the area of the shaded portion bounded by a chord of length 2cm in a circle of radius 1.25 cm (as illustrated below). Please round your answer to 3 decimal places.



17. (6 marks) Solve for θ giving all solutions (in radian measurement) contained in the interval $[0, 2\pi)$.

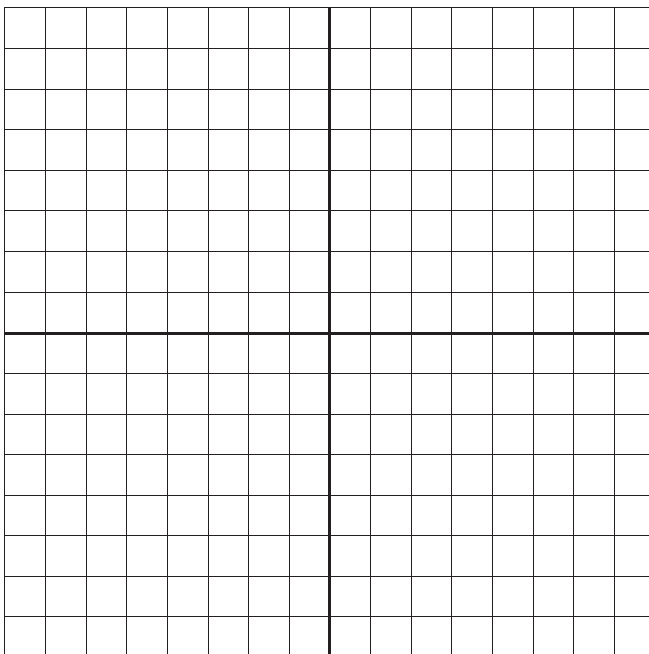
$$2 \sin \theta \cos \theta - \sqrt{3} \cos \theta = 0$$

18. (8 marks) Consider the parabola $f(x) = x^2 - 2x - 3$ and the line $g(x) = -x - 1$

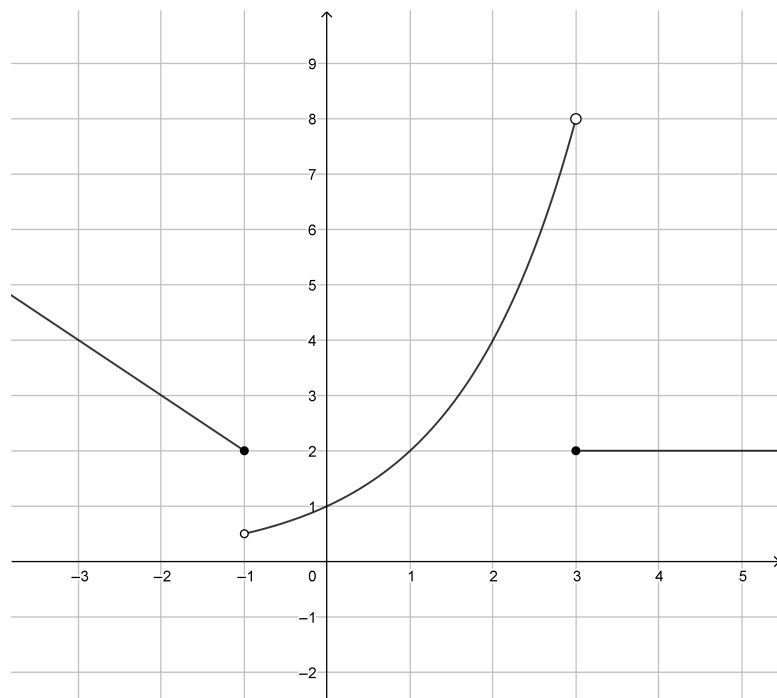
(a) Find the point(s) of intersection between the functions $f(x)$ and $g(x)$.

(b) Sketch the graph of the functions $f(x)$ and $g(x)$ in the space below. Include the following clearly labelled point(s) in your sketch:

- the vertex of the parabola
- any x -intercepts and y -intercepts of both $f(x)$ and $g(x)$
- the point(s) of intersection found in part (a)



19. (5 marks) Consider the graph of the piece-wise function $y = f(x)$ pictured below:



(a) Circle the formula that correctly defines $f(x)$:

$$(i) f(x) = \begin{cases} -x + 1 & x \leq -1 \\ 2^x & -1 < x \leq 3 \\ 2 & x > 3 \end{cases} \quad (ii) f(x) = \begin{cases} -x + 1 & x \leq -1 \\ x^2 & -1 < x < 3 \\ 2 & x \geq 3 \end{cases}$$

$$(iii) f(x) = \begin{cases} -x + 1 & x \leq -1 \\ 2^x & -1 < x < 3 \\ 2 & x \geq 3 \end{cases} \quad (iv) f(x) = \begin{cases} -x + 1 & x < -1 \\ x^2 & -1 < x < 3 \\ 2 & x > 3 \end{cases}$$

(b) Give the value of the following:

$$f(-1)$$

$$f(0)$$

$$f(3)$$

$$f(2)$$

Memory Aid for Remedial Activities for Secondary V Mathematics (201-015-RE)¹

Trigonometric Functions

$$\sin \theta = y/r \quad \csc \theta = r/y$$

$$\cos \theta = x/r \quad \sec \theta = r/x$$

$$\tan \theta = y/x \quad \cot \theta = x/y$$

$$\sin \theta = O/H \quad \csc \theta = H/O$$

$$\cos \theta = A/H \quad \sec \theta = H/A$$

$$\tan \theta = O/A \quad \cot \theta = A/O$$

Trigonometric Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

Inverse Trigonometric Functions (range)

$$\sin^{-1} \text{ or arcsin} : -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\cos^{-1} \text{ or arccos} : 0 \leq \theta \leq \pi$$

$$\tan^{-1} \text{ or arctan} : -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Trigonometric Laws

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 + b^2 - 2ab \cos C = c^2$$

$$b^2 + c^2 - 2bc \cos A = a^2$$

$$a^2 + c^2 - 2ac \cos B = b^2$$

Sectors of Circles

$$\theta = \frac{s}{r}, \quad A = \frac{1}{2}\theta r^2$$

¹Last updated: May 10, 2022