Remedial IV

Final Exam

#1) (5 marks) Expand and simplify:

$$(2x-1)(3x^2-x+1)$$

Solution:

$(2x-1)(3x^2-x+1) = 6x^3 - 2x^2 - 1 - (3x^2 - x + 1)$

$= 6x^3 - 2x^2 - 1 - 3x^2 + x - 1$

$= 6x^3 - 5x^2 + x - 2$

#2) (5 marks) Write this as a single positive power of x:

$$\frac{(x^{-2}x^4)^{-1}}{x^{-5}}$$

Solution:

$\frac{(x^{-2}x^4)^{-1}}{x^{-5}} = \frac{x^5}{x^{-2}x^4}$

$= \frac{x^2x^5}{x^4}$

$= \frac{x^7}{x^4}$

$= x^3$

#3) (5 marks) Divide and simplify:

$$\frac{(x-1)(x+1)}{(x^2-4)} \div \frac{x-1}{x-2}$$

Solution:

$$\frac{(x-1)(x+1)}{(x^2-4)} \div \frac{x-1}{x-2} = \frac{(x-1)(x+1)}{(x^2-4)} \times \frac{x-2}{x-1}$$

$$= \frac{(x-1)(x+1)}{(x-2)(x+2)} \times \frac{x-2}{x-1}$$

$$= \frac{(x+1)}{(x+2)}$$

#4) (5 marks) Subtract and write as a single fraction:

$$\frac{x}{x-1} - \frac{2x}{x+1}$$

Solution:

LCD: $(x-1)(x+1)$

Write each fraction over the LCD:

$$\frac{x}{x-1} = \frac{x(x+1)}{(x-1)(x+1)} = \frac{x^2+x}{x^2-1}$$

$$\frac{2x}{x+1} = \frac{2x(x-1)}{(x+1)(x-1)} = \frac{2x^2-2x}{x^2-1}$$

Finally: $\frac{x}{x-1} - \frac{2x}{x+1} = \frac{x^2+x-(2x^2-2x)}{(x-1)(x+1)} = \frac{-x^2+3x}{x^2-1}$

#5) (6 marks) Factor completely:

a) $x^2(x+3) - 4(x+3)$
b) $x^2 + 2x - 3$

Solution:

a) Factoring out the greatest common factor gives:
$x^2(x+3) - 4(x+3) = (x+3)(x^2-4)$
Next, by the difference of two squares identity:
$(x+3)(x^2-4) = (x+3)(x-2)(x+2)$

c) Using the sum-product method, we find: $x^2 + 2x - 3 = (x+3)(x-1)$

#6) (4 marks) If $f = a + (n-1)d$, find n in terms of f, a, and d.

Solution:

We'll isolate n one step at a time.

We have $f = a + (n-1)d$

$= a + nd - d,$

So, $f - a + d = nd$, or:

$$n = \frac{f - a + d}{d}$$

#7) (5 marks) Solve the inequalities:

$3 \leq 1 - 2x \leq 4$

Solution:

Subtract 1 from each side and get:

$3 \leq 1 - 2x \leq 4 \Leftrightarrow 2 \leq -2x \leq 3$

Divide each side by -2 and reverse the inequalities, with x isolated:

$2 \leq -2x \leq 3 \Leftrightarrow -1 \geq x \geq -\frac{3}{2}$

That is: $x \in [-1.5, -1]$

#8) (5 marks) Solve the system:

$$\begin{cases} 2x & -3y & = 1 & (1) \\ 3x & -2y & = 1 & (2) \end{cases}$$

Solution:

$$\begin{cases} 2x & -3y & = 1 & (\times 3) \\ 3x & -2y & = 1 & (\times 2) \end{cases} \Leftrightarrow \begin{cases} 6x & -9y & = 3 & (1') \\ 6x & -4y & = 2 & (2') \end{cases}$$

The same coefficients are aligned for x.

Now: $(1') - (2') \Leftrightarrow -9y + 4y = 1 \Leftrightarrow -5y = 1 \Leftrightarrow y = -\frac{1}{5} = -0.2$

Substituting in (2), we find:

$3x + \frac{2}{5} = 1 \Leftrightarrow 3x = 1 - \frac{2}{5} = 0.6 \Leftrightarrow x = 0.2$

Hence the solution:

$$(x, y) = (0.2, -0.2)$$

#10) (5 marks) Rationalize the denominator and simplify:

$$\frac{1+\sqrt{3}}{1-\sqrt{3}}$$

Solution:

Multiply through by $1 + \sqrt{3}$:

$\frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{(1+\sqrt{3})^2}{1-3} = -\frac{(1+\sqrt{3})^2}{2}$

Simplify the numerator:

$-\frac{(1+\sqrt{3})^2}{2} = -\frac{1+2\sqrt{3}+3}{2} = -\frac{4+2\sqrt{3}}{2} = -(2+\sqrt{3})$

#11) (10 marks) Solve the equations:

a) $\frac{1}{x-1} + \frac{1}{x} = 2$

b) $\sqrt{x^2+1} - x = 2$

Solution:

a) LCD: $(x-1)(x+1)$

Write each fraction over the LCD:

$\frac{1}{x-1} = \frac{(x+1)}{(x-1)(x+1)}$

$\frac{1}{x+1} = \frac{(x-1)}{(x+1)(x-1)}$

$2 = \frac{2(x^2-1)}{(x-1)(x+1)}$

So, $\frac{1}{x-1} + \frac{1}{x+1} - 2 = \frac{2x-2(x^2-1)}{(x-1)(x+1)} = 2\frac{-x^2+x+1}{(x-1)(x+1)} = -2\frac{x^2-x-1}{x^2-1}$

The numerator has $\Delta = 1+4 = 5$, so, $x = \frac{1\pm\sqrt{5}}{2}$, both valid solutions

b) Isolate the radical:

$\sqrt{x^2+1} = x+2$

Square:

$x^2+1 = (x+2)^2 = x^2+4x+4$

Subtract and simplify:

$4x+3 = 0$

Solve:

$x = -3/4$

Check: $\sqrt{\frac{9}{16}+1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = -\frac{3}{4}+2$: valid solution.

#12) (10 marks) Solve the equations (give your final answers with 2 decimal places):

a) $\log_3(x-1) = 2$

b) $2^{2x-1} = 5$

Solution:

a) $\log_3(x-1) = 2 \Leftrightarrow x-1 = 3^2 \Leftrightarrow x = 10$

b) $2^{2x-1} = 5 \Leftrightarrow 2x-1 = \log_2(5) \Leftrightarrow 2x = \frac{\log(5)}{\log(2)} + 1 \Leftrightarrow x = \frac{1}{2}\left[\frac{\log(5)}{\log(2)} + 1\right] =$ 1.6609

#13) (5 marks) A cashier has $36 in dollars and quarters. If the number of dollar coins is half the number of quarters, how many dollar coins and quarters are in the cash register?

Solution:

Let x and y be the number of dollar coins and quarters, respectively.

The total amount is:

$x + 0.25y = 36$

Moreover, the number of dollar coins is half the number of quarters, so: $x = \frac{1}{2}y$

Hence the system:

$$\begin{cases} x & +0.25y & = 36 & (1) \\ x & -0.5y & = 0 & (2) \end{cases}$$

Subtracting (2) from (1) gives:

$$0.75y = 36 \Leftrightarrow y = \frac{36}{0.75} = 48,$$

and $x = \frac{y}{2} = 24.$

We have 24 dollar coins and 48 quarters.

#14) (5 marks 2-1-1) A telephone company charges a monthly flat rate for the first 200 min of distance calls, and a fixed cost for each minute over the limit.

We know that 300 min of distance calls were billed $150, and 250 min were billed $112.5.

a) How much is the fixed cost?
b) How much is the flat rate?
c) How long is the distance communication (in minutes) for a $300 bill

Solution:

a) We paid $112.5 for 250 min and $150 for 300 min. So, each additional minute above 200 min costs:

$$m = \frac{150 - 112.5}{300 - 250} = 0.75\ \$$$

That is the fixed cost.

b) If x is the length of a distance call, with $x \geq 200$, and y is the corresponding amount charged, then:

$$\frac{y - 150}{x - 300} = m \Leftrightarrow y = f(x) = 0.75(x - 300) + 150 = 0.75x - 75$$

The flat rate is $f(200) = \$75$

c) $y = 300 \Leftrightarrow 0.75x - 75 = 300 \Leftrightarrow 0.75x = 300 + 75 = 225 \Leftrightarrow x = \frac{375}{0.75} = 500$ min

#15) (5 marks) Determine the equation of the line that passes through the points: $(1,1)$ and $(-3,2)$. Give the slope of the line and its intercepts.

Solution:

Slope: $m = \frac{2-1}{-3-1} = -0.25$

Equation in point-slope form:

$y - 1 = -0.25 * (x - 1) \Leftrightarrow y = -0.25x + 1.25$

y-intercept: $(0,1.25)$

x-intercept: $-0.25x + 1.25 = 0 \Leftrightarrow x = 5$. The x-intercept is: $(5,0)$

#16) (5 marks) Draw a sketch of the parabola whose equation is:

$$y = x^2 - 3x + 2$$

(Show the coordinates of the vertex and the intercepts)

Solution:

Vertex:

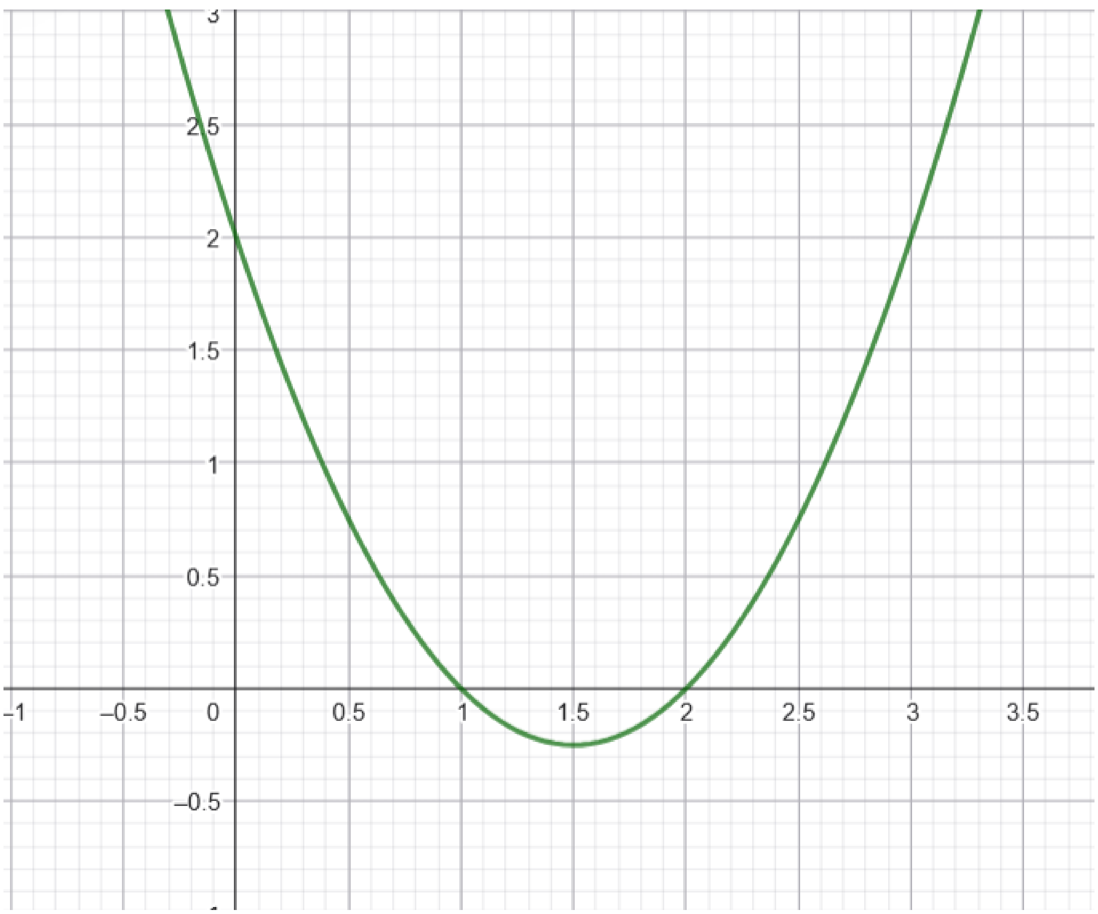
$x_v = \frac{3}{2}, y_v = -\frac{1}{4}$

$V = (1.5, -0.25)$

y-intercept: $(0,2)$

$y = (x-1)(x-2)$, the x-intercepts are: $(1,0), (2,0)$

Graph:



#17) (5 marks) Let $f(x)$ be the function: $f(x) = \frac{x^2-3x+2}{x^2-2}$

a) For what value(s) of x do we have: $f(x) = 0$?
b) Give the domain of f.
c) Evaluate $f(1)$ and $f(-1)$.

Solution:

a) $f(x) = 0 \Leftrightarrow x^2 - 3x + 2 = 0 \Leftrightarrow x = 1, x = 2$
b) f is defined for all the values that do not cancel the denominator, that is for all x except $-\sqrt{2}, \sqrt{2}$
c) $f(1) = 0, f(-1) = -6$

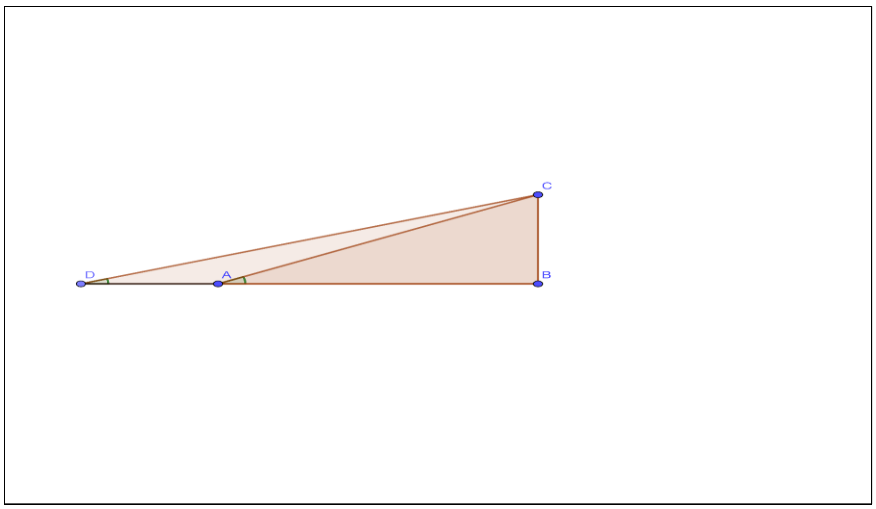
#18 (5 marks 3-2) A right triangle has a hypotenuse of length 17 and one leg of length 15.

a) Find the length of the other leg.
b) Find the area of the triangle.

Solution:

a) By Pythagoras, $x^2 + 15^2 = 17^2$, so: $x^2 = 64$, or: $x = 8$
b) $area = \frac{1}{2} * 8 * 15 = 60$

#19) (5 marks) In the figure below, the distance between A and D is $1\ ft$ and the angles BAC and BDC are $60\ °$ and $30\ °$ respectively. Find the distance between B and C.



Solution:

Put $y = DB, x = AB, z = CB$

Then:

$\frac{y}{z} = \cot(30°)$, $\frac{x}{z} = \cot\,(60°)$, and $y - x = 1$

Hence, $\frac{y-x}{z} = \frac{1}{z} = \cot(30) - \cot\,(60)$, or: $z = \frac{1}{\cot(30) - \cot\,(60)} = \sqrt{3}/2$ ft