DAWSON COLLEGE
Mathematics Department

FINAL EXAMINATION

Calculus I (201-103-DW)
Fall 2019

Instructors:
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Student Name: ______________________________

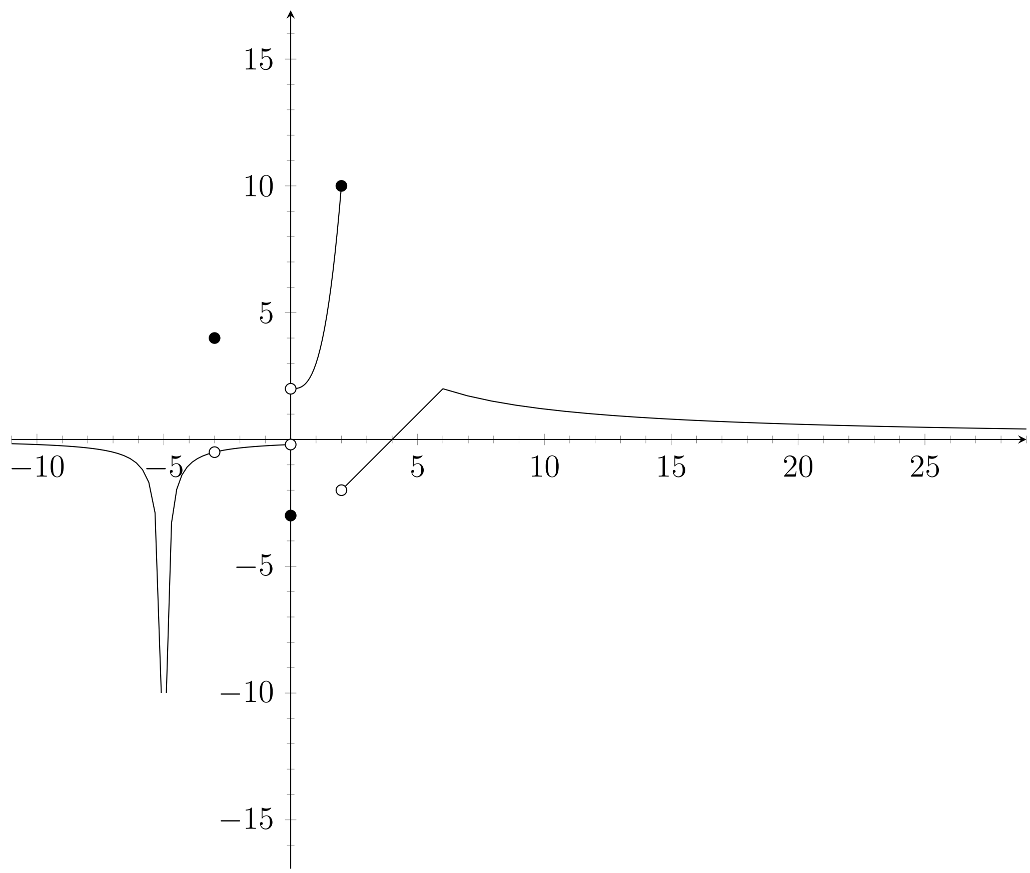
Student ID. #: ______________________________

Instructions:

- Print your name and ID number in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly and indicate your final answer.
- Only calculators Sharp EL 531.X/ XG/XT are permitted.
- This examination booklet must be returned intact.

This examination consists of 13 questions. Please ensure that you have a complete examination booklet before starting.

(1) **(5 marks)** The graph of $y = f(x)$ is given below. Answer the following questions: (Use DNE if it doesn't exist and ∞ or $-\infty$ if needed.)



(a) $\lim_{x \to 0^+} f(x) =$

(b) $\lim_{x \to 2^-} f(x) =$

(c) $\lim_{x \to -3} f(x) =$

(d) $\lim_{x \to 2} f(x) =$

(e) $\lim_{x \to 6} f(x) =$

(f) $\lim_{x \to +\infty} f(x) =$

(g) $\lim_{x \to -5^-} f(x) =$

(h) $f(0) =$

(i) Vertical Asymptote(s):

(j) Horizontal Asymptote(s):

(2) **(2+5+5 marks)** Evaluate the following limits if they exist. (show your work and write ∞ or $-\infty$ if needed.)

(a) $$\lim_{x \to -2^+} \frac{1 - x^2}{x^2 - 4}$$

(b) $$\lim_{x \to -1} \frac{\dfrac{1}{x+4} - \dfrac{1}{3}}{x^2 - 1}$$

(c) $$\lim_{x \to 2} \frac{3x^2 - x - 10}{2x^2 + 3x - 14}$$

(3) **(6 marks)** Let $f(x) = \begin{cases} 2x^2 + 3x & \text{if } x \leq 0, \\ \dfrac{x}{x-1} & \text{if } 0 < x < 2, \\ \ln(x-1) & \text{if } 2 \leq x \end{cases}$

(a) Find $\lim_{x \to 2} f(x)$ if it exists.

(b) Find $\lim_{x \to 0} f(x)$ if it exists.

(c) Find the value(s) of x for which the function is discontinuous.

(4) **(5 marks)** Use **only the limit definition of derivative** (4-step process) to find $f'(x)$ where $f(x) = 3x^2 + 4x - 1$.

(5) **(16 marks)** Find the derivative of the following functions. You do NOT need to simplify them algebraically.

(a) $y = \dfrac{(3x+1)^5}{\sqrt{x^2-3x}}$

(b) $f(x) = (\tan^2 x)e^{(3x^2-5x+3)}$

(c) $y = \sqrt[3]{\arcsin\sqrt{x} + 3\pi^2}$

(d) $y = (4x^2-1)^{\cos x}$

(6) **(6 marks)** Find $\frac{dy}{dx}$ in the given relation below and then find **an equation of the tangent line** to the graph of this relation at $(1, 0)$.

$$3xe^y - 6y = x^2 - 4$$

(7) **(6 marks)** Find the absolute maximum value and the absolute minimum value of the function on the given interval.

$f(x) = x^4 - 2x^2 \quad$ on $[-2, 0]$

(8) **(6 marks)** Find the second derivative of $f(x) = \dfrac{\ln x}{x^2}$ and simplify. Then evaluate $f''(e)$.

(9) **(4 marks)** The amount of digital information created globally, t months after the beginning of 2018 is approximately

$$f(t) = 4000\left(\frac{t}{12} + 1\right)^{1.09} \qquad 0 \leq t \leq 36$$

billion gigabytes.

How fast was the amount of digital information changing at the beginning of 2019?
Note that t is in month!

(10) **(1+4 marks)** The quantity x of a certain product demanded each week is related to the unit price p by the equation

$$p = \frac{5000}{0.01x^2 + 1} \qquad 0 \leq x \leq 20$$

(a) Find the revenue function.

(b) Use the marginal revenue function to estimate the revenue realized from producing and selling the10th unit.

(11) **(6 marks)** Two ships leave the same port at noon. Ship A sails north at 20 km/h and Ship B sails east at 16 km/h. How fast is the distance between the ships increasing at 1:30 p.m.?

Hint: At 1:30 p.m. Ship A is 30 km and Ship B is 24 km away from the port.

(12) Let $$f(x) = \frac{x}{1-x^2}.$$

You also have its first and second derivatives as $f'(x) = \dfrac{1+x^2}{(1-x^2)^2}$ and $f''(x) = \dfrac{2x^3+6x}{(1-x^2)^3}$.

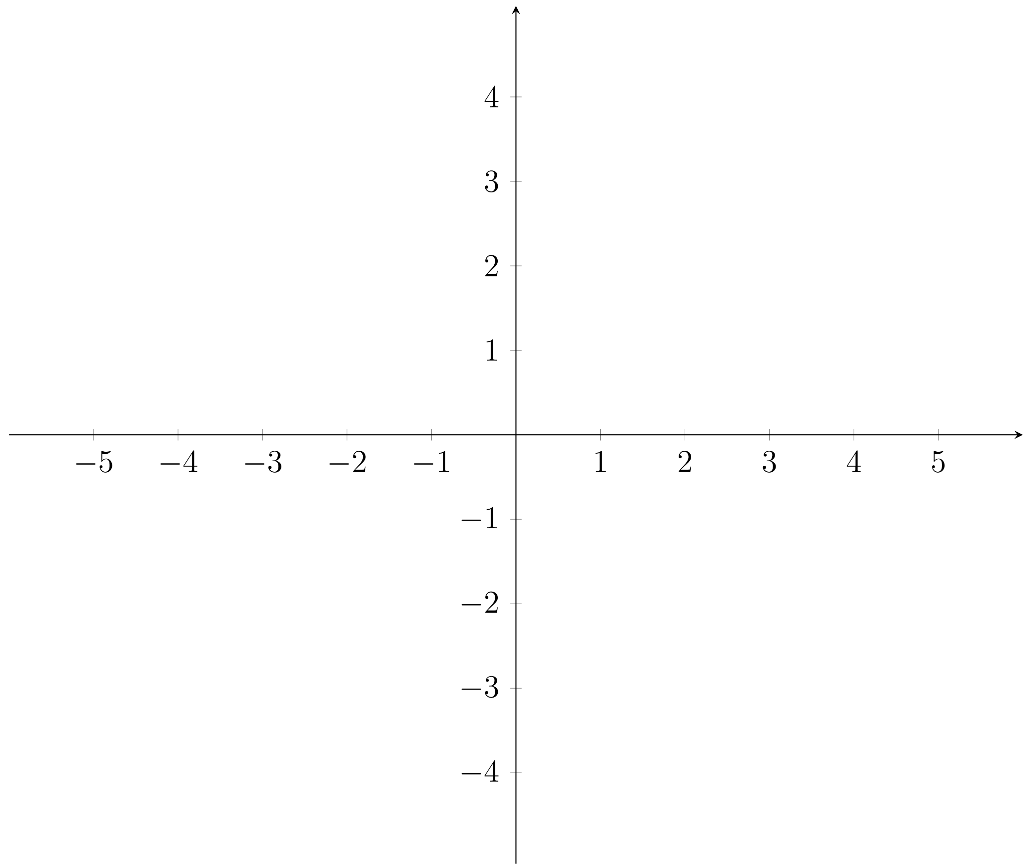
(a) **(2 marks)** Find the x-intercept(s) and the y-intercept.

(b) **(3 marks)** Find the horizontal and vertical asymptotes.

(c) **(4 marks)** Find the intervals where the function is increasing and the intervals where it is decreasing and its relative extrema.

(d) **(4 marks)** Determine where the function is concave up, where it is concave down and find the inflection point(s).

(e) **(4 marks)** Use your answers from parts (a), (b), (c) and (d) to sketch the graph of the function.



(13) **(6 marks)** A closed rectangular box is to have a rectangular base whose length is twice its width and a volume of 1152 cm^3. If the material for the base and the top costs 0.80$/$cm^2$ and the material for the sides costs 0.20$/$cm^2$. Determine the dimensions of the box that can be constructed at minimum cost. (Justify your answer!)