DAWSON COLLEGE

Department of Mathematics

Final Examination

Calculus I

201-103-DW

Winter 2022

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Note:

- Write your answer in the space provided for each question in this examination paper.
- Use the reverse sides if needed, or for rough work (indicate this clearly).
- The exam has 12 pages (12 questions). It must be returned intact.

Reserved for Marking

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
/12	/13	/5	/12	/6	/6	/6	/6	/8	/15	/5	/6

1. [12 marks] Evaluate each of the following limits (show all your steps):

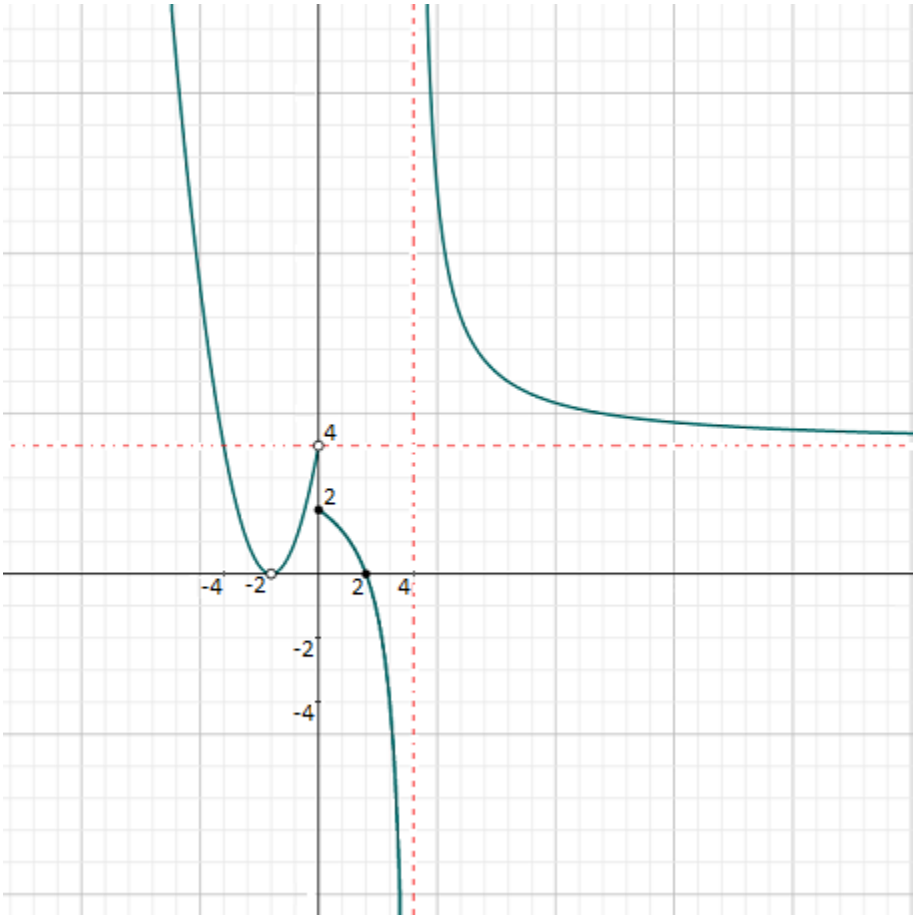
(a) $\lim_{x \to -3} \dfrac{x^2 + 6x + 9}{2x^2 + 3x - 9}$

(b) $\lim_{x \to 0} \dfrac{\sqrt{16 - x} - 4}{7x}$

(c) $\lim_{x \to -\infty} \dfrac{x^2 + 4x + 4}{4x^2 - 4}$

(d) $\lim_{x \to 4} \dfrac{\dfrac{1}{x+1} - \dfrac{1}{5}}{x - 4}$

2. [13 marks] Consider the graph below, representing a function $f(x)$.



(a) Find the following, if they exist.

i) $\lim_{x \to -2^-} f(x)$

ii) $\lim_{x \to -2^+} f(x)$

iii) $\lim_{x \to 0} f(x)$

iv) $\lim_{x \to 2} f(x)$

v) $\lim_{x \to 4} f(x)$

vi) $\lim_{x \to -\infty} f(x)$

vii) $\lim_{x \to \infty} f(x)$

viii) The interval(s) where both $f'(x) > 0$ and $f''(x) > 0$

ix) The interval(s) where $f'(x)$ is decreasing

x) The interval(s) where $f'(x) < 0$

(b) Find all points of discontinuity of the function. Justify your answers using the definition of continuity (and using correct notation).

3. [5 marks] Let $f(x) = 7x - x^2$. Find $f'(x)$ using only the limit definition of the derivative.

4. [12 marks] Find y'; you do not have to simplify the answer algebraically:

(a) $y = 5x \tan(x^3)$

(b) $y = \dfrac{\sqrt{2x-4}}{(4x+2)^3}$

(c) $y = \arctan(4x) - \sin^4 x$

(d) $y = x^{x^2-5x}$

5. [6 marks] Find $f''(2)$ given $f(x) = e^{3x^2-5x}$.

6. [6 marks] Find the values of x at which the function $f(x) = (x^2 - 3x)^5$ has horizontal tangent lines.

7. [6 marks] Find the equation of the tangent line to the graph of the relation

$$3x^4 + 5xy - 3y^2 = 1$$

at the point $(1, 2)$.

8. [6 marks]The unit price p (in dollars) and the quantity demanded x (in units of one thousand) for a product are related by

$$p + x^2 = 2800$$

How fast is the quantity demanded increasing when the unit price is \$300 and it is dropping at a rate of \$10 per week?

9. [8 marks] A company determines that their cost of production (in dollars) is given by:

$$C(x) = 5000 + 40x$$

while their selling price (also in dollars) is:

$$p = 200 - 0.1x$$

where x is the number of units sold.

a) Find the marginal profit and use it to estimate the profit made on the 201st unit.

b) Find the exact profit made on the 201st unit.

10. [15 marks] Consider the function:

$f(x) = \dfrac{12x - 12}{x^2}$ with $f'(x) = \dfrac{-12x + 24}{x^3}$ and $f''(x) = \dfrac{24x - 72}{x^4}$.

(a) Find its x- and y- intercepts.

(b) Find its asymptotes, if any. Justify your answer using limits.

(c) Find the intervals where f is increasing and where f is decreasing, and local extrema, if any.

(d) Find the intervals where f is convex and where f is concave, and points of inflection, if any.

(e) Graph the function and label the important points.

11. [5 marks] Find the absolute extrema of $f(x) = x^3 - 12x$ on $[-1, 3]$.

12. [6 marks] A rectangular area is to be enclosed and divided into three equal parts by two dividers. The outside fence costs $\$10$ per running foot installed, and the dividers cost $\$20$ per running foot installed. If $\$600$ is available to spend for the fencing material, what are the dimensions that will maximize the area enclosed?