DAWSON COLLEGE
MATHEMATICS DEPARTMENT

Final Examination
Winter 2016

Calculus 2 (201-203-DW)

Date: Thursday, May 19th, 2015 at 2pm
Instructors: C. Farnesi, A. Jimenez, A. Juhasz

1. **[5 marks]** Find the average value of the function
$$g(x)=\frac{7\sqrt{x^3}+5x^2-8}{\sqrt{x}}$$
over the interval $[1,4]$.

2. **[5 marks]** The number of Montreal CEGEP students vacationing abroad each summer between 2010 and 2020 is expected to model the function $f'(x)=28(x+11)^{3/4}$ where x is measured in years, and $x=0$ corresponds to the summer of 2010. The number of Montreal CEGEP students who vacationed abroad in the summer of 2015 was 3125. What is the projected number of Montreal CEGEP students vacationing abroad in 2020?

3. **[7 marks]** Use the limit definition of the definite integral (Riemann Sums) to evaluate
$$\int_0^5 (10x-x^3)dx$$
No marks will be given for using the rules of anti-differentiation.

4. **[6 marks]** Find the area of the region completely enclosed by the graphs of $f(x)=x^2-8x$ and $g(x)=-x$.

5. **[6 marks]** Each month, the quantity demanded x (in units of a hundred) of a certain commodity is related to the unit price p (in dollars) by the demand function $D(x)=60+2x-x^2$, and the supply function $S(x)=2x^2+11x-60$. Find the producers' surplus if the unit market price is set at equilibrium.

6. **[5 marks]** Use Simpson's Rule with $n=4$ to approximate to 4 decimal places the value of the definite integral
$$\int_1^9 (3-\ln x)dx$$

7. **[20 marks]** Solve the following integrals:

a. $\int \left(-8\cos\frac{x}{5} + 3\sin 6x\right) dx$

b. $\int \frac{8x}{(5-2x)^5} dx$

c. $\int (5+4x)(\ln(5+2x))dx$

d. $\int \frac{-5x^2+4x-21}{(x+2)(x^2+3)} dx$

8. **[5 marks]** Evaluate the limit

$$\lim_{x\to 1} \frac{x^3 - 2\cos(x-1) + e^{3-3x}}{-1+\sin(1-x)+e^{x-1}}$$

9. **[6 marks]** Evaluate the integral, if it converges:

$$\int_4^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

10. **[6 marks]** Use separation of variables to find the particular solution of the differential equation $y' = 6x - 2xy$ subject to the initial condition $y(3) = 4$.

11. **[7 marks]** Find the fourth Taylor Polynomial of the function $f(x) = 3\ln x$ at $x = 1$, and then use it to estimate the value of $f(1.1) = 3\ln 1.1$

12. **[7 marks]** Show that the following series converges, and then find its sum:

$$\sum_{n=0}^{\infty} \left(\frac{2(-1)^n}{4^n} + \frac{(3)^n}{5^{n-1}}\right)$$

13. **[15 marks]** Determine if each of the following series is convergent or divergent. State the test used.

a) $\sum_{n=2}^{\infty} \frac{3n^5-7n+2}{16n+8n^3-n^5}$

b) $\sum_{n=1}^{\infty} \frac{1}{3+\sqrt{n^3}}$

c) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Information Page

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$C.S. = \int_0^{\bar{x}} D(x)dx - \bar{p}\bar{x}$$

$$P.S. = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)dx$$

$\int_a^b f(x)dx \cong \frac{\Delta x}{2}\big(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\big)$ where $\Delta x = \frac{b-a}{n}$

$\int_a^b f(x)dx \cong \frac{\Delta x}{3}\big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \cdots + 4f(x_{n-1}) + f(x_n)\big)$
where $\Delta x = \frac{b-a}{n}$ and n is even

$$P_n(x) = f(a) + f'(a) \cdot (x-a) + \frac{f''(a)}{2!} \cdot (x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!} \cdot (x-a)^n$$

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Answers

1. Average value is $\frac{197}{6}$
2. 4373 students
3. -31.25
4. $57.167\ units^2$
5. $\$30{,}416.67$
6. 12.2705
7. A) $-40\sin\frac{x}{5} - \frac{1}{2}\cos 6x + C$
   B) $\frac{5}{2(5-2x)^4} - \frac{2}{3(5-2x)^3} + C$
   C) $(5x + 2x^2)\ln(5 + 2x) - x^2 + C$
   D) $-7\ln|x + 2| + \ln|x^2 + 3| + C$
8. 17
9. $2e^{-2}$
10. $-\ln|3 - y| = 9 - x^2\ \ or\ y = 3 - e^{9-x^2}\ \ or\ any\ other\ variation$
11. $P_4(x) = 3(x-1) - \frac{3}{2}(x-1)^2 + (x-1)^3 - \frac{3}{4}(x-1)^4\ \ and\ \ f(1.1) \sim P_4(1.1) = 0.285925$
12. By separating the series into 2 different ones: the first has $|r| = \frac{1}{4} < 1$ and the second has $|r| = \frac{3}{5} < 1$. Since they both converge, so does the original series. To find the total sum of the series, $S = S_1 + S_2 = \frac{8}{5} + \frac{25}{2} = 14.1$
13. A) diverges by the divergence test
    B) converges by the comparison test (to p-series)
    C) diverges by the integral test