Dawson College
Mathematics Department

Calculus-II
201-203-DW
Winter - 2019

Friday, May 17, 2019
9:30 am – 12:30 pm

Student Name: ______________________________

Student I. D. #: ______________________________

Instructor Name: ______________________________

Instructors: C. Farnesi, I. Rajput, A. Jimenez, A. Hindawi

20-Marks

Q-1) Find the following integral;

a) $\int 3x^5\left(x^3-1\right)^4 dx$

b) $\int \frac{x+2}{x\left(1+9x^2\right)} dx$

c) $\int \frac{\ln x}{x^2} dx$

d) $\int_1^4 \frac{\cos\left(\frac{2\pi}{x}\right)}{x^2} dx$

6-Marks

Q-2) Find the average value of the function $f(x) = x\, e^{9-x^2}$ on the interval $[1,3]$.

6-Marks

Q-3) *Zunera Baby* line of products estimates that the daily marginal cost function associated with producing fancy baby dresses is $C'(x) = 0.006x^2 - 0.01x + 5$, and the daily fixed cost incurred in producing these dresses is $\$150$. What is the total cost in producing the first 200 dresses?

6-Marks

Q-4) The demand equation for a certain make of toys is given by $p = 160 - 0.2x^2$, where p is the unit price in dollars and x is the quantity demanded in units of a hundred. The supply function for these toys is given by $p = 4x$, where p is the unit price in dollars and x stand for the number of toys that the supplier will put on the market, in units of a hundred. Determine

a) the market equilibrium.

b) the consumers' surplus at market equilibrium

5-Marks

Q-5) Find the limit: $\displaystyle\lim_{x\to 0^+} \frac{8 + (x-2)^3 - 3\sin 4x}{3x - e^{3x} + \cos 2x}$

6-Marks

Q-6) Use the limit definition of the definite integral (Riemann sums) to evaluate $\displaystyle\int_0^3 \left(12x^3 - 5x + 7\right) dx$.

6-Marks

Q-7) Find the area of the region completely enclosed by the graphs of the functions

$$f(x) = 2x^2 - 5x + 5 \;\; and \;\; g(x) = x^2 + 2x - 1$$

5-Marks

Q-8) Evaluate the improper integral: $\displaystyle\int_{e^2}^{\infty} \frac{dx}{x(\ln x)^3}$

5-Marks

Q-9) Use separation of variables to solve the differential equation $e^{2y}\, y' = \sin(x+\pi)$, subject to the initial condition $y = 0 \;\; when \;\; x = \pi$. Leave your answer in implicit form.

5-Marks

Q-10) Use the Simpson's rule with $n = 4$ to approximate the value of the definite integral. Round the final answer in three decimal places. $\displaystyle\int_1^6 e^x \ln|x|\, dx$

5-Marks

Q-11) Find the sum of the convergent series $\displaystyle\sum_{n=1}^{\infty} \frac{4 + 3^n}{4^{n+1}}$

5-Marks

Q-12) Find the third Taylor Polynomial of the function $f(x) = e^{2x} - \sin(3x) + 7$ at $x = 0$.

5-Marks

Q-13) Find the limit of the sequence to check its convergence or divergence $\left\{ \dfrac{\ln(n+2)}{\sqrt{n}} \right\}$.

15-Marks

Q-14) Test the following series for convergence or divergence. State the test used.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{4n}{n^3 + 7}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^2}{2n^2 + 3n + 1}$

c. $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$

Name: ____ **I.D.#________________________**

You must include this sheet in your booklet when you return your exam.

INFORMATION PAGE

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$CS = \int_0^{\bar{x}} D(x)\,dx - \bar{p}\cdot\bar{x} \quad or \quad CS = \int_0^{\bar{x}} \left[D(x) - \bar{p}\right]dx$$

$$PS = \bar{p}\cdot\bar{x} - \int_0^{\bar{x}} S(x)\,dx \quad or \quad PS = \int_0^{\bar{x}} \left[\bar{p} - S(x)\right]dx$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx = \tfrac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + f(x_n)\right] \text{ where } \Delta x = \tfrac{b-a}{n}$$

Simpson's Rule

$$\int_a^b f(x)\,dx = \tfrac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3)\ldots 4f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \tfrac{b-a}{n}$ and n is even

Taylor polynomial

$$P_n(x) = f(a) + f'(a)\cdot(x-a) + \tfrac{f''(a)}{2!}\cdot(x-a)^2 + \tfrac{f'''(a)}{3!}\cdot(x-a)^3 + \ldots + \tfrac{f^n(a)}{n!}\cdot(x-a)^n$$

If $ax^2 + bx + c = 0 \ (a \neq 0)$ *then* $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Answers

Q-1/a) $\frac{(x^3-1)^6}{6}-\frac{(x^3-1)^5}{5}+C$

Q-1/b) $2\ln|x|-\ln\left|1+9x^2\right|+\frac{1}{3}\arctan(3x)+C$

Q-1/c) $-\frac{\ln x+1}{x}+C$

Q-1/d) $-\frac{1}{2\pi}$

Q-2) $\frac{e^8-1}{4}$

Q-3) $\$16,950$

Q-4) a) $(20\ ,\ \$80)$ b) $\$106,666.67$

Q-5) $\frac{12}{13}$

Q-6) $\frac{483}{2}$

Q-7) $\frac{125}{6}\,(unit)^2$

Q-8) $\frac{1}{8}$

Q-9) $\frac{1}{2}e^{2y}=-\cos(x+\pi)+\frac{3}{2}$

Q-10) 648.743

Q-11) $\frac{13}{12}$

Q-12) $P_3(x)=8-x+2x^2+\frac{35}{6}x^3$

Q-13) $\lim_{n\to\infty}\frac{\ln(n+2)}{\sqrt{n}}=0,\quad Convergent$

Q-14/a) *Convergent by Comparision Test*

Q-14/b) *Divergent by nth term test*

Q-14/c) $\sum_{n=3}^{\infty}\frac{\ln n}{n}$ *Divergent by Integral Test*