Dawson College

Mathematics Department

Final Examination

201-203-DW Calculus-II Social Science

Winter - 2022

May 25, 2022 14:00-17:00

Student Name: _____

Student I.D. #: _____

Instructor Name: _____

Instructors: A. Gambioli, I. Rajput, O. Veres

INSTRUCTIONS:

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer, use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 15 questions on 11 pages, including this cover page and a formula sheet on page 11.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

1. [5 marks] If $f'(x) = \dfrac{x\sqrt[3]{x} - 2x + 4}{x}$ and the graph of f passes through the point $\left(1, \frac{3}{4}\right)$ then find $f(x)$.

2. [15 marks] Evaluate the integrals

   a. $\displaystyle\int \left(\frac{3}{\sqrt{1-x^2}} + \frac{5}{1+4x^2} \right) dx$

b. $\int \frac{7x^2 - 6x + 12}{(x-2)(x^2+3)}\,dx$

c. $\int 4x^3(x^2+3)^7\,dx$

3. [7 marks] Find the area of the region bounded by the functions $f(x)=x^2-1$ and $g(x)=7-x^2$

4. [5 marks] Find the average value of $f(t) = e^{\cos(2t)} \sin(2t)$ over $[0, \ \pi/2]$

5. [7 marks] Use the limit definition (Riemann Sums) of the definite integral to evaluate
$\int_1^2 (4x - 3x^2)dx$

6. [7 marks] The supply function for a certain product is $p = \frac{120x}{500 - x^2}$ where p is the unit wholesale price in dollars and x stands for the quantity that will be made available in the market by the supplier, measured in units of hundreds. Determine the producers' surplus if the wholesale market price is set at \$24/unit.

7. [5 marks] The rate of change in the number N of international travelers in millions to a country from 2010 through 2015 can be modeled by

   $N'(t) = 1.234t - 2.6$, where t is the year, with t=0 corresponding to 2010.

   Find the net change in the number of travelers from 2010 to 2015

8. [5 marks] Find

$$\lim_{x\to 1}\left(\frac{3x}{\ln x}-\frac{3}{x\ln x}\right)$$

9. [7 marks] Evaluate the improper integral

$$\int_{-\infty}^{0}\frac{6}{(9-x)^{3/2}}\,dx$$

10. [7 marks] Solve the separable differential equation. (Do not isolate y)

$$\frac{\sec^2(y)}{x}y' = 2\ln x$$

11. [5 marks] Use the Simpson's rule with $n = 4$ to approximate the values of the following definite integral. Round your answer to three decimal places.

$$\int_1^3 \frac{e^x}{x}\,dx$$

12. [5 marks] Find the third Taylor polynomial of the function $\boldsymbol{f(x)=\ln(2x+1)}$ at $x=0$.

13. [4 marks] Find the limit of the sequence to check its convergence or divergence.

$$\left\{\frac{e^{2n}+3n}{5e^{2n}+4n}\right\}_{n\geq 1}$$

14. [6 marks] Find the sum of the convergent series

$$\sum_{n=1}^{\infty}\frac{5+2^{n-1}}{7^{n+1}}$$

15. [15 marks] Test the convergence or divergence of the following series. State the test used.

a. $\sum_{n=2}^{\infty} \frac{2n-1}{\sqrt{n^6+3}}$

b. $\sum_{n=1}^{\infty} \frac{(n+1)(2n+1)}{10n^2+8}$

c. $\sum_{n=e}^{\infty} \frac{\sqrt{\ln n}}{2n}$

Answers:

1. $f(x) = \frac{3}{4}x^{3/4} - 2x + 4\ln(x) + 2$
2. $a.\ \ 3\arcsin x + \frac{5}{2}\arctan(2x) + C, \quad b. \quad 4\ln|x-2| + \frac{3}{2}\ln|x^2+3| + C,$
   $c.\ \ \frac{2}{9}(x^2+3)^9 - \frac{3}{4}(x^2+3)^8 + C$
3. $\frac{64}{3}u^2$
4. $\frac{1}{\pi}(e - e^{-1})$
5. -1
6. \$383,434
7. 2,425,000
8. 6
9. 4
10. $\tan y = x^2 \ln x - \frac{x^2}{2} + C$
11. 8.641
12. $P_3(x) = 2x - 2x^2 + \frac{8}{3}x^3$
13. $\frac{1}{5}$
14. $\frac{31}{210}$
15. $a.\ Convergent, by\ Comparison\ Test, \quad b.\ Divergent, by\ Test\ for\ Divergence$
    $c.\ Divergent, by\ Integral\ Test$