DAWSON COLLEGE
MATHEMATICS DEPARTMENT
Applied Mathematics for Computer Science

201-920-DW
Winter 2025
Final Examination
May 28th, 2025
Time Limit: 3 hours

Name: Solutions

ID#:

Instructors: M. Hitier, A. Jimenez

- ▷ This exam contains 12 pages (including this cover page) and 15 problems. Check to see if any pages are missing.
- ▷ Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- ▷ Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner.
- ▷ You are only permitted to use the **Sharp EL-531**** calculator.
- ▷ This examination booklet must be returned intact.
- ▷ Good luck!

Question	Points	Score
1	5	
2	5	
3	5	
4	15	
5	5	
6	7	
7	9	
8	9	
9	5	
10	4	
11	10	
12	5	
13	5	
14	5	
15	6	
Total:	100	

[5 pts] 1. Simplify the following, expressing the answers with positive exponents only:

$$\left(\frac{x^2y^3}{z^4}\right)^0 \cdot \sqrt[3]{\frac{xy^3}{(x^{-2}y^5)^4yz^{-1}}}$$

$$= 1 \cdot \left(\frac{x y^3}{(x^{-2} y^5)^4 y z^{-1}}\right)^{1/3}$$

$$= \left(\frac{x y^3}{x^{-8} y^{20} y z^{-1}}\right)^{1/3}$$

$$= \left(\frac{x^9 \cdot z}{y^{18}}\right)^{1/3} = \boxed{\frac{x^3 z^{1/3}}{y^6}}$$

[5 pts] 2. Divide by long division to find the quotient and remainder $\dfrac{3x^4 + 3x^2 + 8x - 1}{x^2 - x + 2}$.

$$\begin{array}{r} 3x^2 + 3x \\ x^2 - x + 2 \,\overline{)\, 3x^4 + 0x^3 + 3x^2 + 8x - 1} \\ -(3x^4 - 3x^3 + 6x^2) \\ \hline 3x^3 - 3x^2 + 8x - 1 \\ -(3x^3 - 3x^2 + 6x) \\ \hline 2x - 1 \end{array}$$

quotient: $3x^2 + 3x$

Remainder: $2x - 1$

[5 pts] 3. Noting clearly the intercepts, the vertex, the axis of symmetry, and the range, graph

$$y = -x^2 + 2x + 15$$

x-intercepts: $0 = -x^2 + 2x + 15$

$0 = x^2 - 2x - 15$

$0 = (x-5)(x+3)$

↳ $\boxed{(5,0) \ \& \ (3,0)}$

y-intercept: $\boxed{(0,15)}$

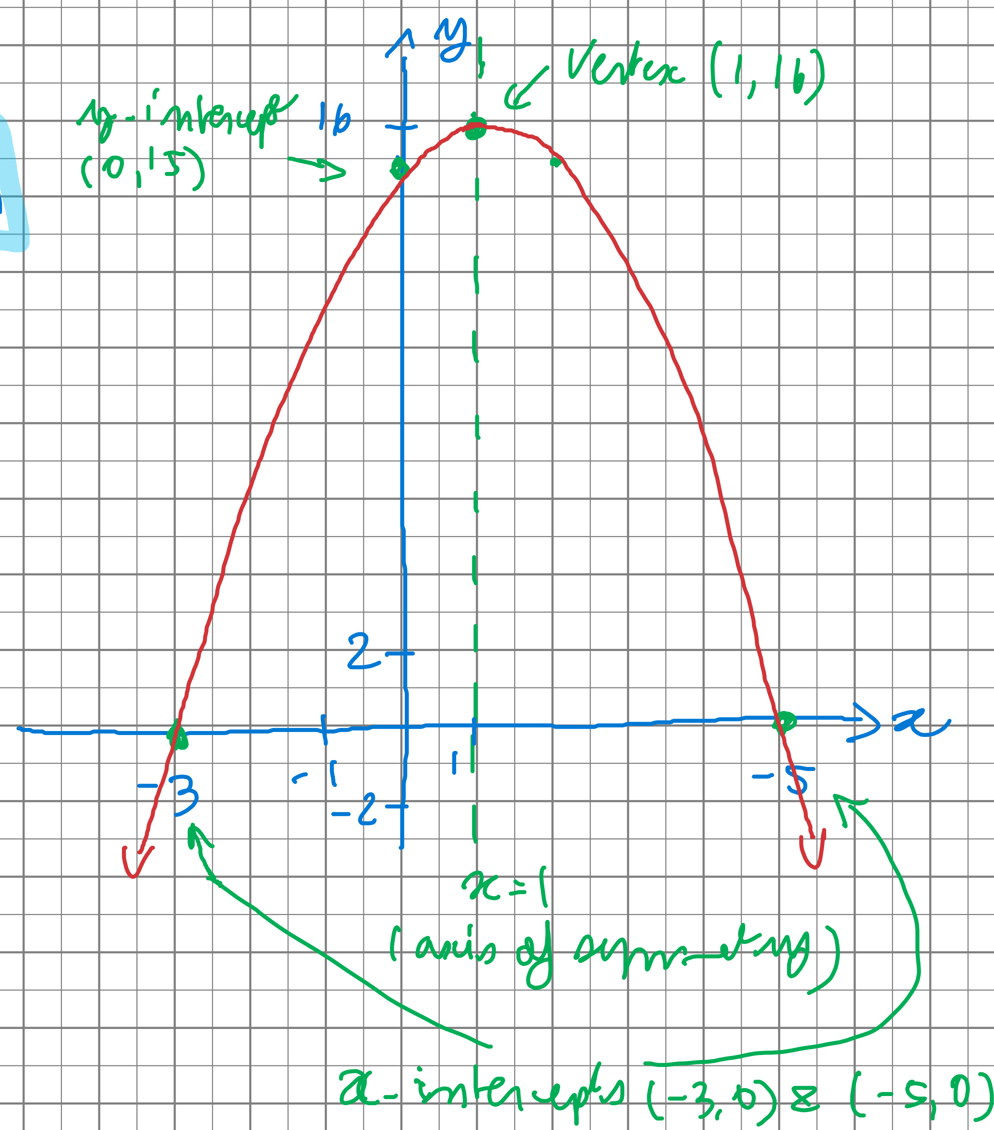
axis of symmetry: $x = \frac{-2}{-2}$

$\boxed{x = 1}$

Vertex: $-1^2 + 2(1) + 15 = 16$

↳ $\boxed{(1,16)}$

Range: $\boxed{[-\infty, 16]}$



[15 pts] 4. Solve for x

(a) $9x^3 = 6x^2 + 2x$

$$9x^3 - 6x^2 - 2x = 0$$

$$x(9x^2 - 6x - 2) = 0$$

$$\boxed{x = 0} \qquad 9x^2 - 6x - 2 = 0$$

$$\Delta = (-6)^2 - 4 \cdot 9(-2) = 36 + 72$$

$$= 108 = 6^2 \cdot 3$$

$$x = \frac{6 \pm 6\sqrt{3}}{2 \cdot 9} = \boxed{\frac{1 \pm \sqrt{3}}{3}}$$

(b) $\sqrt{x-8} + \sqrt{x} = 2$

$$\left(\sqrt{x-8}\right)^2 = \left(2 - \sqrt{x}\right)^2$$

$$\cancel{x} - 8 = 4 - 4\sqrt{x} + \cancel{x}$$

$$\frac{-12}{-4} = \frac{-4\sqrt{x}}{-4}$$

$$(3)^2 = \left(\sqrt{x}\right)^2$$

$$9 = x$$

check: $\sqrt{9-8} + \sqrt{9} = 1 + 3 = 4 \neq 2$

$\Rightarrow$ $\boxed{\text{No solution.}}$

(c) $\dfrac{x}{x-1} + \dfrac{1}{x} = \dfrac{x^2+1}{\underbrace{x^2-1}_{(x-1)(x+1)}}$

LCD: $x(x-1)(x+1)$

$$\frac{x\,(x(\cancel{x-1})(x+1))}{\cancel{x-1}} + \frac{\cancel{x}(x-1)(x+1)}{\cancel{x}} = \frac{(x^2+1)\,x(\cancel{x-1})(\cancel{x+1})}{(\cancel{x-1})(\cancel{x+1})}$$

$$x^2(x+1) + x^2 - 1 = x(x^2+1)$$

$$\cancel{x^3} + x^2 + x^2 - 1 = \cancel{x^3} + x$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$\boxed{x = -1/2}$ $\qquad x = 1$ ← extraneous

[5 pts] 5. Solve for x and give the solution graph and set. $\qquad -4 < 3 - 2x \leq 1$

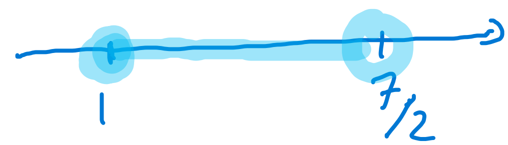
$-3 \quad -3 \quad -3$

$-7 < -2x \leq -2$

$\div -2$

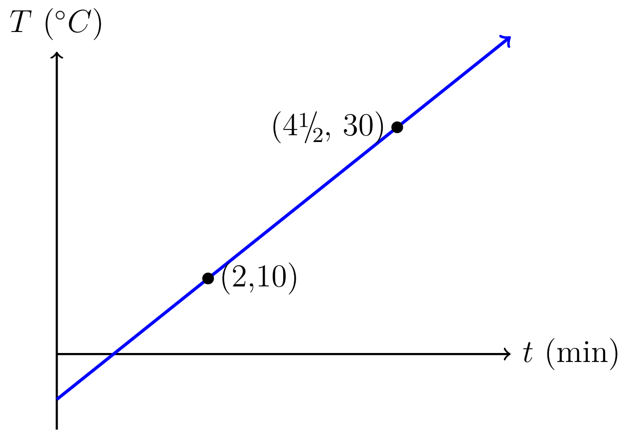
$\boxed{\frac{7}{2} > x \geq 1}$

Graph:



Set: $[1, \frac{7}{2})$.

6. Vilmer took a slice of pizza from the freezer and put it in the oven.
The pizza's temperature, T (in degrees Celsius) as a linear function of time, t (in minutes), is graphed.



[5 pts] (a) Find the linear equation for the relationship.

[2 pts] (b) What was the temperature of the freezer?

(a) Slope: $\dfrac{30-10}{4.5-2} = \dfrac{20}{2.5} = 8$

Equation: $\boxed{T - 10 = 8(t-2)}$
or $\boxed{T = 8t - 6}$

(b) The temperature of the freezer is the initial temperature: $\boxed{-6°C}$

7. Given $f(x) = \dfrac{1}{x+1}$ and $g(x) = 2x^2 + 1$

[2 pts] (a) State the domain of $f(x)$

[2 pts] (b) Determine the range of $g(x)$

[5 pts] (c) Find $(g \circ f)(x)$ and simplify as much as possible.

(a) $D_f = \boxed{\mathbb{R} \setminus \{-1\}}$

(b) $x^2 \geqslant 0 \Rightarrow 2x^2 \geqslant 0 \Rightarrow 2x^2 + 1 \geqslant 1$

$R_g = \boxed{[1, \infty)}$

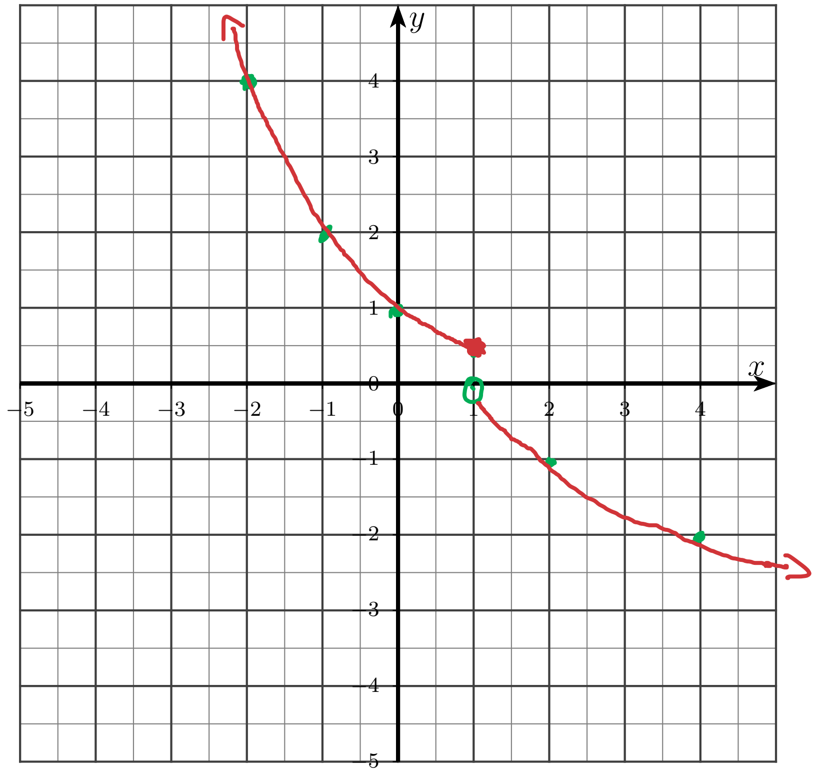
(c) $(g \circ f)(x) = g(f(x)) = 2\left(\frac{1}{x+1}\right)^2 + 1$

$= 2\frac{1}{(x+1)^2} + \frac{(x+1)^2}{(x+1)^2}$

$= \frac{2 + x^2 + 2x + 1}{(x+1)^2}$

$= \boxed{\frac{x^2 + 2x + 3}{(x+1)^2}}$

[5 pts] 8. (a) Using some well chosen points, graph the function $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x & \text{if } x \leq 1 \\ -\log_2 x & \text{if } x > 1 \end{cases}$



Then

[2 pts] (b) State its domain $\mathbb{R}$

[2 pts] (c) State its range $(-\infty, 0) \cup [\frac{1}{2}, \infty)$

x	y
-2	4
-1	2
0	1
1	$1/2$
2	-1
4	-2

$-\log_2(1) = 0$

[5 pts] 9. Write in terms of the simplest logarithm possible

$\log\left(\frac{10\sqrt{x}}{x^2-1}\right) = \log 10x^{1/2} - \log[(x-1)(x+1)]$

$= \log 10 + \log x^{1/2} - (\log|x-1| + \log|x+1|)$

$= \boxed{1 + \frac{1}{2}\log x - \log|x-1| - \log|x+1|}$

[4 pts] 10. Use your calculator to evaluate $\log_2 3$

Show all relevant intermediate steps and give your answer with a precision of 3 decimal places.

$\log_2 3 = \frac{\ln 3}{\ln 2} \approx \boxed{1.585}$

[10 pts] 11. Solve the following equations

(a) $\ln(x-3)+\ln(x-5)-\ln 3=0$

$\ln(x-3)(x-5)=\ln 3$

$(x-3)(x-5)=3$

$x^2-5x-3x+15=3$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$\boxed{x=6}$ $\quad x=2$ ← extraneous

Check:

$x=6$:

$\ln(6-3)+\ln(6-5)-\ln 3$

$=\ln 3+0-\ln 3=0$ ✓

$x=2$ $\ln(2-3)$ DNE ✗

(b) $\dfrac{1}{9^x}=3^{x+2}$

$\dfrac{1}{3^{2x}}=3^{x+2}$

$3^{-2x}=3^{x+2}$

$-2x=x+2$

$-3x=2$

$\boxed{x=-\frac{2}{3}}$

[5 pts] 12. Verify the following identity

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 2\csc^2\theta$$

$$\underline{LHS} = \frac{1-\cos\theta}{(1+\cos\theta)(1-\cos\theta)} + \frac{1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cancel{\cos\theta}+1+\cancel{\cos\theta}}{1^2-\cos^2\theta} = \frac{2}{\cancel{\cos^2\theta}+\sin^2\theta-\cancel{\cos^2\theta}}$$

$$= \frac{2}{\sin^2\theta} = 2\csc^2\theta = RHS$$

[5 pts] 13. Solve the following trigonometric equation $2\sin^2(\theta) - \sin(\theta) = 0$ for $0 \leq x < 2\pi$

$$\sin\theta(2\sin\theta - 1) = 0$$

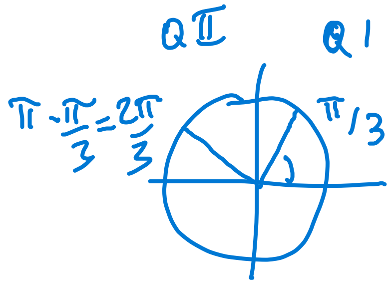
$\sin\theta = 0$

$\boxed{\theta = 0, \pi}$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

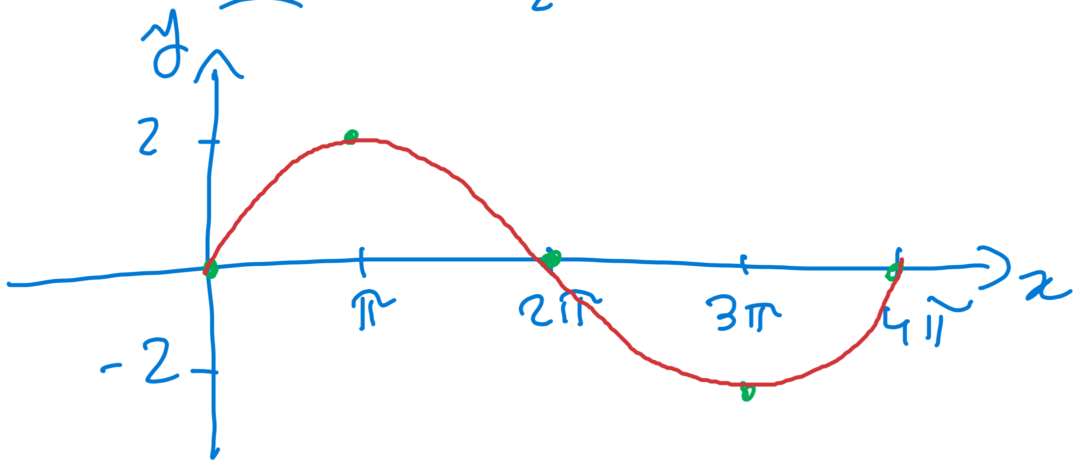
$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}}$



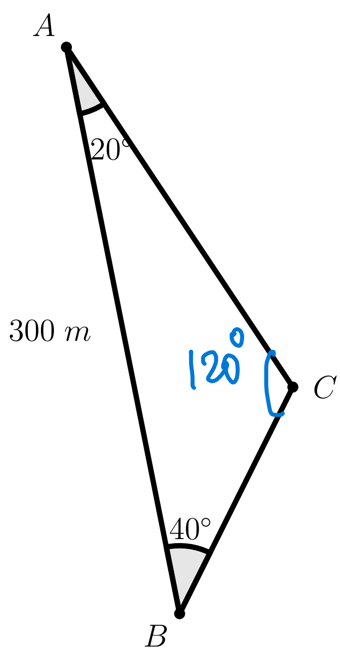
[5 pts] 14. Graph the following function $y = 2\sin\left(\frac{x}{2}\right)$ over one period.

Amplitude . 2

Period: $2\pi \div \frac{1}{2} = 4\pi$



[6 pts] 15. Estimate AC and BC (give your answer to 3 decimal places).



$\hat{C} = 180° - 40° - 20°$

$$\frac{AC}{\sin 40°} = \frac{300}{\sin 120°}$$

$$AC = \frac{300 \sin 40°}{\frac{\sqrt{3}}{2}} = \frac{600 \sin 40°}{\sqrt{3}}$$

$$\approx \boxed{222.668}$$

$$\frac{BC}{\sin 20°} = \frac{300}{\sin 120°}$$

$$BC = \frac{300 \sin 20°}{\sin 120°}$$

$$\approx \boxed{118.479}$$