Dawson College,
Mathematics Department,
Fall Semester 2023,
January, 8-th, 2024, 9:30-12:30.

Final Exam 201-923-DW,
Section 1.

Instructor: S. Soltuz.

Student Name: SOLUTIONS

Student I.D. #: ________________________

Instructor Name: ________________________

INSTRUCTIONS:

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 12 questions on 13 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

Q-1) (10 Marks) Solve the system by using Gaussian Elimination

$$\left\{ \begin{array}{l} x + 2y + z = 1 \\ x + y \qquad = 0 \\ \quad y + 2z = 3 \end{array} \right\}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{array}\right] \simeq \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 2 & 3 \end{array}\right] \cdot (-1) \simeq$$

$R_2 = R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{array}\right] \simeq \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_3 = R_3 - R_2$

$\underline{z = 2}$

$y + z = 1 \qquad y = 1 - z = 1 - 2 = -1 \quad , \underline{y = -1}$

$x + 2y + z = 1$

$x - 2 + 2 = 1 \qquad \underline{x = 1}$

Q-2) (6=3+3 Marks) (Two questions) Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix},$$

$\cancel{A \cdot B}$

a) compute, if possible, $A \cdot B$ and $B \cdot A$,
b) find B^{-1} and compute $B \cdot B^{-1}$.

$$B \cdot A = \begin{bmatrix} 7 & 1 & -3 \\ -2 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{-1} \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$$

$$\underline{B \cdot B^{-1} = I_2} \checkmark$$

Q-3) (8=2+4+2 Marks) (Three questions) Let

$$\left\{ \begin{array}{c} x + 2y = 7 \\ 3x + 5y = 18 \end{array} \right\},$$

a) write the system in matricial form, that is Ax=b
b) compute the inverse by using the inversion algorithm $[A, I]$...
c) use the inverse matrix to compute the solution.

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 18 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right] \simeq \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right] \simeq$$

$R_2 - 3R_1$ $\quad$ $-R_2$

$$\simeq \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$R_1 - 2R_2$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 18 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$x = 1$

$y = 3$

Q-4) (10 Marks) Compute the determinant

$$\begin{vmatrix} 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2$$

$$\det A = 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2-1 = 1$$

Q-5) (10 Marks) Find the inverse matrix A^{-1}, by using the adjoint matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad A_{12} = -\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad A_{13} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$A_{21} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -1$$

$$A_{31} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = 2 \quad A_{33} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$$

Q-6) (10 Marks) Given $(0,1)$, $(1,5)$, $(-1,-1)$, find the second degree funtion ($f(x) = Ax^2 + Bx + C$) that will pass through all these given points.

$$\begin{cases} 1 = 0 + 0 + C \\ 5 = A + B + C \\ -1 = A - B + C \end{cases}$$

$$\begin{cases} \underline{C = 1} \\ A + B = 4 \\ A - B = -2 \end{cases}$$

$$2A = 2$$

$$\underline{A = 1}$$

$$B = 4 - A = 4 - 1 = 3$$

$$f(x) = x^2 + 3x + 1$$

Q-7) (6=3+3 Marks) (TWO QUESTIONS) Given

$$\bar{u} = (-1, 2),$$
$$\bar{v} = (-3, 1),$$

find a) $2\bar{u} - 3\bar{v}$
b) $\|\bar{u}\|$

$$2\bar{u} - 3\bar{v} = (-2, 4) + (9, -3) = (7, 1)$$

$$\|\bar{u}\| = \sqrt{1+4} = \sqrt{5}$$

Q-8) (10 Marks) Solve for x

$$\log(x+1) = 1 - \log(x-2).$$

$$\log(x+1) + \log(x-2) = 1$$

$$\log[(x+1)(x-2)] = 1$$

$$x^2 - x - 2 = 10$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0 \rightarrow 4$$

$$\rightarrow \cancel{-3} \text{ reject}$$

Q-9) (8 Marks) Solve for x

$$3^{5x} = 2^{x+1}.$$

$$\ln \Big| \quad 3^{5x} = 2^{x+1}$$

$$5x \ln 3 = (x+1) \ln 2$$

$$5x \ln 3 = x \ln 2 + \ln 2$$

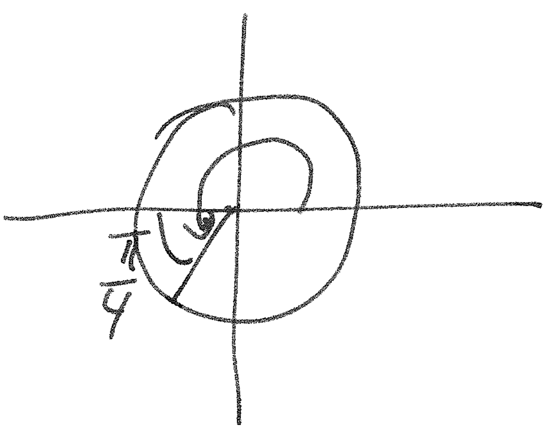
$$x(5\ln 3 - \ln 2) = \ln 2$$

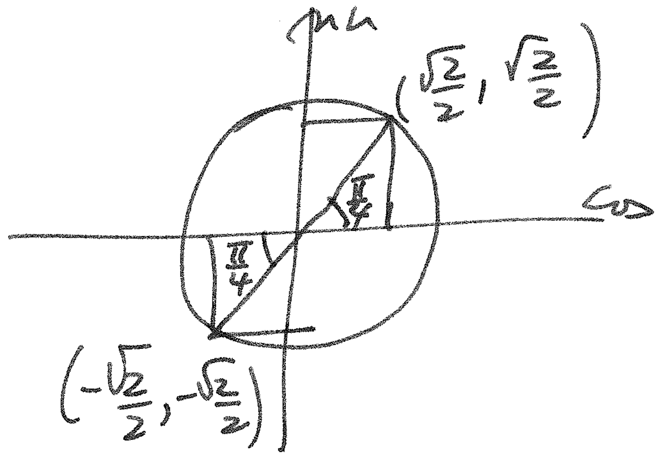
$$\underline{x = \frac{\ln 2}{5\ln 3 - \ln 2}}$$

Q-10) (6 Marks) Find the exact value of

$$\sin \frac{5\pi}{4}$$

$$\sin \frac{5\pi}{4} = \sin\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$





Q-11) (8 Marks) Find the exact value of

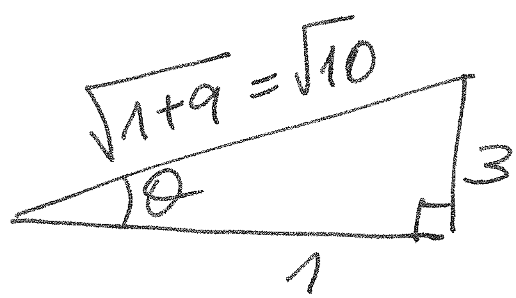
$$\sin(\arctan 3)$$

$$\sin(\underbrace{\arctan 3}_{\theta}) = \sin\theta = \frac{3}{\sqrt{10}}$$

$$\theta = \arctan 3$$

$$\tan\theta = 3$$

$$\tan\theta = \frac{3}{1}$$



Q-12) (8 Marks) Solve for $\theta \in [0, 2\pi[$

$$\sec\theta - 2 = 0$$

$$\sec\theta = 2$$

$$\frac{1}{\cos\theta} = \frac{2}{1}$$

$$\cos\theta = \frac{1}{2} \quad , \quad \theta_1 = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

$$\underline{\theta_2 = -\frac{\pi}{3}}$$