Dawson College,
Mathematics Department,
Fall Semester 2025,
Dec. 18-th, 14:00-17:00.

Final Exam 201-923-DW Applied Math. for Chem. Tech.
Section 1.

Instructor: S. Soltuz.

SOLUTIONS

Student Name:______________________

Student I.D. #: ______________________

Instructor Name: ______________________

INSTRUCTIONS:

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 18 questions on 19 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

Q-1) (5 Marks) Solve for x:

$$12 - 7\ ((-2)\ (1-x)+x) = -\,(3x+10)$$

$$12-7\cdot(-2+2x+x)=-3x-10$$

$$12-7\cdot(-2+3x)=-3x-10$$

$$12+14-21x=-3x-10$$

$$-21x+3x=-12-14-10$$

$$-18x=-36$$

$$x=\frac{-36}{-18}$$

$$\underline{x=2}$$

Q-2) (6=3+3 Marks) (Two independent questions)

(a) Find the equation of the line through $(-1,-4)$ and $(2,-1)$.

(b) Find the equation of the line through the point $(1,-2)$, and parallel to

$$2x-5y=3.$$

a

$$m=\frac{-1-(-4)}{2-(-1)}=\frac{-1+4}{2+1}$$

$$m=\frac{3}{3}=1$$

$$y=x+n$$

$$-1=2+n \qquad n=-3$$

$$\underline{y=x-3}$$

b

$$2x-5y=3$$

$$2x-3=5y$$

$$y=\frac{2}{5}x-\frac{3}{5}$$

$$m=\frac{2}{5}$$

$$y=\frac{2}{5}x+n$$

$$-2=\frac{2}{5}\cdot 1+n$$

$$\frac{-10-2}{5}=n$$

$$n=\frac{-12}{5}$$

$$\underline{y=\frac{2}{5}x-\frac{12}{5}}$$

Q-3) (5 Marks) Find K such that the two lines are perpendicular

$$Kx - 2y = 3,$$
$$x + 2y = 7.$$

- $Kx - 3 = 2y \Rightarrow y = \frac{K}{2}x - \frac{3}{2}$
- $2y = -x + 7 \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$

$\frac{K}{2} \cdot \left(-\frac{1}{2}\right) = -1$ $\qquad$ $\underline{K = 4}$

Q-4) (6 Marks) Use Gauss-Jordan Elimination (i.e. reduce until you will receive the Identity matrix I_2), to solve the system:

$R_2 = R_2 - 3R_1$ $\left\{ \begin{array}{c} x + 4y = 9 \\ 3x + 2y = 7 \end{array} \right\}$

$$\left[\begin{array}{cc|c} 1 & 4 & 9 \\ 3 & 2 & 7 \end{array}\right] \simeq \left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & -10 & -20 \end{array}\right] \div(-10)$$

$$\left[\begin{array}{cc|c} 1 & 4 & 9 \\ 0 & 1 & 2 \end{array}\right] \simeq \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

$R_1 = R_1 - 4R_2$

$$\left\{ \begin{array}{l} X = 1 \\ y = 2 \end{array} \right.$$

Q-5) (6 Marks) Find the quadratic function $f(x) = Ax^2 + Bx + C$ that passes through $(1, 2)$, $(-1, 2)$, $(0, 1)$.

$$\begin{cases} A + B + C = 2 \\ A - B + C = 2 \\ C = 1 \end{cases} \qquad \begin{cases} A + B = 1 \\ A - B = 1 \\ C = 1 \end{cases}$$

$$2A = 2 \Rightarrow A = 1$$

$$B = 0$$

$$C = 1$$

$$\underline{f(x) = x^2 + 1}$$

Q-6) (5 Marks) Write the system of equations,and the matricial form, (DO NOT SOLVE IT) that can be used to balance the chemical reaction

$$x \ \ CaCl_2 + y \ \ Na_3PO_4 \rightarrow z \ \ Ca_3\left(PO_4\right)_2 + w \ \ NaCl$$

Ca $x = 3z$

Cl $2x = w$

Na $3y = w$

P $y = 2z$

O $4y = 8z$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc|c} 0 & 4 & -8 & 0 & 0 \end{array} \end{array}$$

Q-7) (6 Marks) Given

$$f(x) = \frac{1-2x}{2x-1} \quad \text{and} \quad g(x) = \frac{3-4x}{2x+1},$$

simplify completely and find

$$fog = ?$$

I

$$(f\circ g)(x) = f(g(x)) = \frac{1-2g(x)}{2g(x)-1} =$$

$$= \frac{1-2\frac{3-4x}{2x+1}}{2\frac{3-4x}{2x+1}-1} = \frac{\frac{2x+1-6+8x}{2x+1}}{\frac{6-8x-2x-1}{2x+1}} =$$

$$= \frac{\frac{10x-5}{2x+1}}{\frac{-10x+5}{2x+1}} = \frac{10x-5}{-10x+5} = -1$$

II

Remark that

$$f(x) = \frac{1-2x}{-(1-2x)} = -1$$

Q-8) (5 Marks) If $f(x) = -3x + 2$ and $g(x) = -x + B$, find B such that $(fog)(x) = (gof)(x)$.

$(fog)(x) = f(g(x)) = -3(-x + B) + 2$

$= 3x - 3B + 2$

$(gof)(x) = g(f(x)) = -(-3x + 2) + B$

$= 3x - 2 + B$

$\cancel{3x} - 3B + 2 = \cancel{3x} - 2 + B$

$-3B - B = -2 - 2$

$-4B = -4$

$\underline{B = 1}$

Q-9) (5 Marks) Find the inverses, f^{-1} if

$$f(x) = \sqrt{\frac{3x-1}{4}} - 5$$

$$y = \sqrt{\frac{3x-1}{4}} - 5$$

$$y + 5 = \sqrt{\frac{3x-1}{4}}$$

$$(y+5)^2 = \frac{3x-1}{4}$$

$$4(y+5)^2 = 3x - 1$$

$$4(y+5)^2 + 1 = 3x$$

$$x = \frac{4(y+5)^2+1}{3}$$

$$f^{-1}(x) = \frac{4(x+5)^2+1}{3}$$

Q-10) (6 Marks) Consider the function

$$y = y_0 \; e^{kx},$$

that passes through the points

$$\begin{aligned}&(6, 200)\\ &(8, 600)\,,\end{aligned}$$

find the coresopndend k and y_0 in the above curve.

$$\begin{cases} 200 = y_0 \cdot e^{6k} \\ 600 = y_0 \cdot e^{8k} \end{cases} \Big/ \div$$

$$\frac{600}{200} = \frac{e^{8k}}{e^{6k}} \qquad 3 = e^{2k}$$

$$\ln 3 = 2k \;, \quad \boxed{k = \frac{\ln 3}{2}}$$

$$200 = y_0\, e^{6 \cdot \frac{\ln 3}{2}} \;, \quad 200 = y_0\, e^{3 \ln 3}$$

$$200 = y_0\, e^{\ln 3^3} \qquad 200 = y_0\; 27$$

$$\boxed{y_0 = \frac{200}{27}}$$

$$\frac{5}{2} = 2.5 \qquad 2-\frac{1}{2} = \frac{4-1}{2} = \frac{3}{2}$$

Q-11) (8=4+4 Marks) (Two independent questions)
(a) Express as a single logarithm with coefficient of one

$$\frac{5}{2}\log x + \frac{1}{2}\log 2y - \log x^3y^2$$

(b) Express in terms of simple logarithms

$$\log\left(\frac{a^5}{\sqrt{bc^3}}\right)$$

a)

$$\log x^{\frac{5}{2}} + \log(2y)^{\frac{1}{2}} - \log x^3y^2$$

$$= \log \frac{x^{\frac{5}{2}} . 2^{\frac{1}{2}} y^{\frac{1}{2}}}{x^3 y^2} =$$

$$= \log \frac{1}{x^{\frac{1}{2}}} \frac{1}{y^{\frac{3}{2}}} . \sqrt{2} =$$

$$= \log \sqrt{\frac{2}{x y^3}}$$

(b)

$$\log a^5 - \log\sqrt{bc^3} =$$

$$= 5\log a - \frac{1}{2}\log b -$$

$$- \frac{3}{2}\log c$$

Q-12) (5 Marks) If

$$f\left(x\right)=\log_{2}\left(x+7\right)$$

then find x if $f\left(x\right)=3$.

$$\log_2(x+7)=3$$

$$x+7=2^3$$

$$x=2^3-7$$

$$x=8-7=1$$

Q-13) (5 Marks) The hydrogen ion concentration , x, of a solution is 0.002 mol/L. What is the pH of the solution?

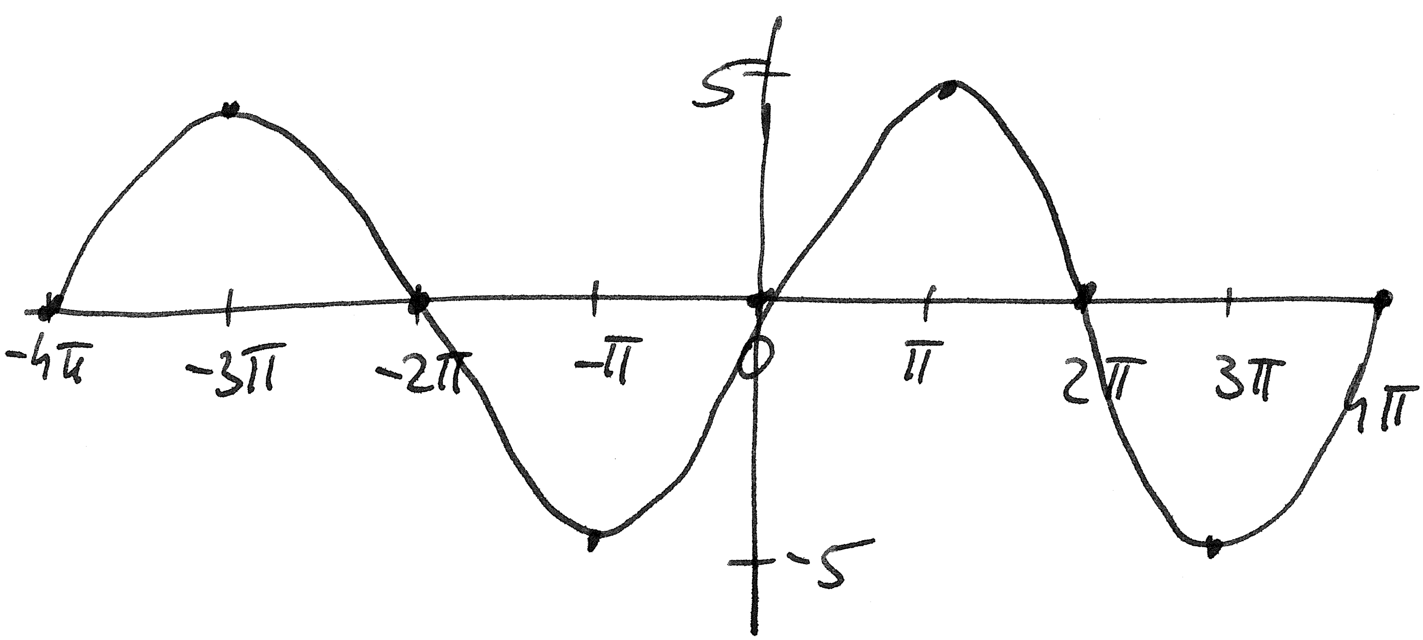
$$pH = -\log(H^+)$$

pH=-log(0.002)=2.69897~2.7

Q-14) (6 Marks) Sketch the graph of the given function over the given domain:

$$y = 5\sin\left(\frac{x}{2}\right), \quad for \quad -4\pi \leq x \leq 4\pi$$

x	-4π	-3π	-2π	$-\pi$	0	π	2π	3π	4π
$y = 5\sin\frac{x}{2}$	0	5	0	-5	0	5	0	-5	0

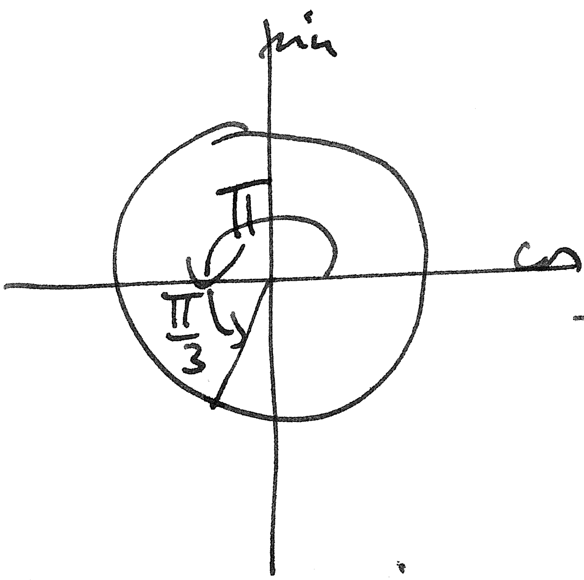


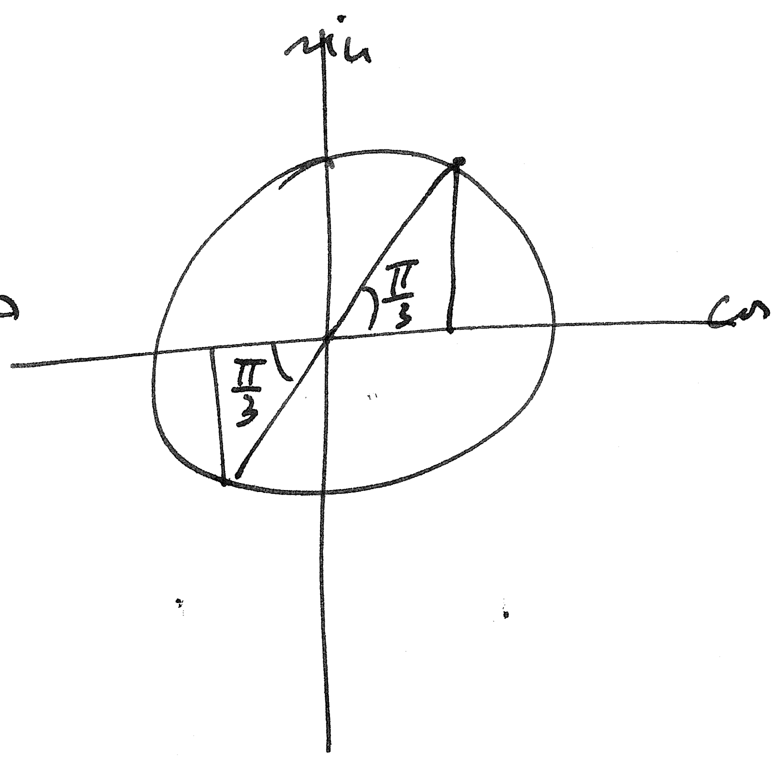
$$\csc\frac{\pi}{2} = \frac{1}{\sin\frac{\pi}{2}} = 1$$

Q-15) (5 Marks) Compute the exact value for

$$\tan\frac{4\pi}{3} + \csc\frac{\pi}{2} = \sqrt{3} + 1$$

$$\tan\frac{4\pi}{3} = \tan\frac{3\pi+\pi}{3} = \tan\left(\pi + \frac{\pi}{3}\right)$$





$$= \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

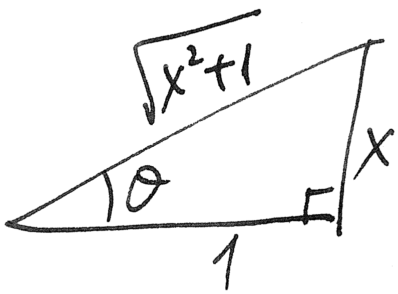
$$\theta = \tan^{-1}\frac{x}{1}$$

$$\tan\theta = \frac{x}{1}$$

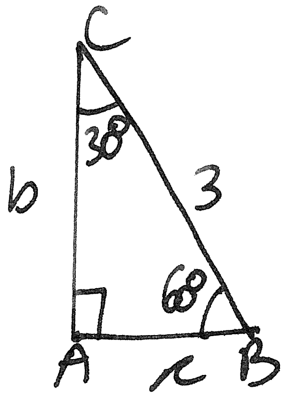
Q-16) (6 Marks) Findt he exact value of

$$\cos\left(\tan^{-1} x\right)$$

$$\cos\left(\tan^{-1}\frac{x}{1}\right) = \cos\theta = \frac{1}{\sqrt{x^2+1}}$$



Q-17) (6 Marks) Solve the right triangle ΔABC, (give the exact values; no marks for approximations), ($\widehat{A} = 90^0$) that has $\widehat{B} = 60^0$ and $a = 3\ cm$.



$$\frac{b}{3} = \sin 60^\circ$$

$$b = 3 \sin 60^\circ = \frac{3\sqrt{3}}{2}$$

$$\frac{c}{3} = \cos 60^\circ$$

$$c = 3 \cos 60^\circ = \frac{3}{2}$$

Q-18) (4 Marks) If $\overline{u} = (3, -2)$, $\overline{v} = (4, 2)$ and $\overline{w} = (-2, 1)$ compute $\|\overline{u} + 2\overline{v} - 3\overline{w}\|$

$$\overline{u} + 2\overline{v} - 3\overline{w} =$$

$$= (3, -2) + (8, 4) - (-6, 3) =$$

$$= (3 + 8 + 6, -2 + 4 - 3) =$$

$$= (17, -1)$$

$$\|(17, -1)\| = \sqrt{17^2 + (-1)^2} = \sqrt{290}$$

$$= 17.02938$$