

FALL 2021 - FINAL EXAMINATION
Dawson College - Mathematics Department
Applied Mathematics (201-923-DW S01)
(LABORATORY TECHNOLOGY - ANALYTICAL CHEMISTRY)

201-923-DW 01
FALL 2021
Final Examination
December 16th, 2021
Time Limit: 3 hours

Name: _____

ID#: _____

Instructor: E. Richer

- This exam contains 13 pages (including this cover page) and 14 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner.
- You are only permitted to use the **Sharp EL-531X, XG or XT** calculator.
- This examination booklet must be returned intact.
- Good luck!

Question	Points	Score
1	7	
2	9	
3	8	
4	8	
5	7	
6	9	
7	10	
8	10	
9	4	
10	7	
11	6	
12	6	
13	4	
14	5	
Total:	100	

1. (7 marks) Consider the function

$$f(x) = \log_3(2x + 1)$$

(a) State the domain of $f(x)$

(b) State the range of $f(x)$

(c) Plot $y = f(x)$ in the space provided below. Include at least 4 different properly labelled points. Make sure that the general shape of the function is clearly illustrated.



2. (9 marks) Consider the following matrices:

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Compute (where possible):

i. $A^T + B$

ii. CD

iii. DE

iv. C^{-1}

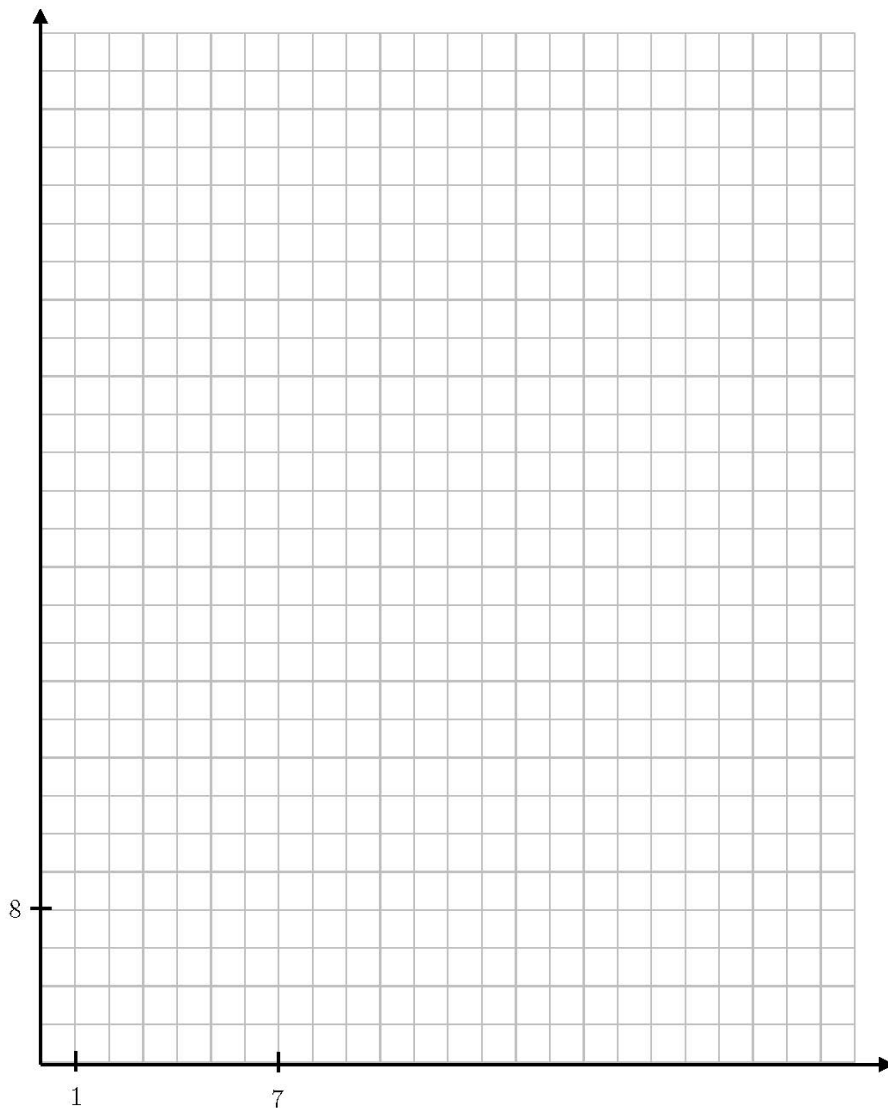
(b) Which of the matrices are in row-echelon form?

(c) Which of the matrices are in reduced row-echelon form?

3. The height of an arch $H(x)$ (in feet) as a function of horizontal distance x (in feet) from one extremity of its base is given by:

$$H(x) = -\frac{3}{8}x^2 + 9x, \quad \text{where } 0 \leq x \leq 24$$

- (a) (4 marks) Plot the function $H(x)$ making sure to include at least 4 properly labelled points. The sketch should clearly indicate any intercepts as well as the vertex.



- (b) (2 marks) What is the highest point of the arch?
- (c) (2 marks) What is the height of the arch 8 feet from its centre?

4. Let $A = \begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix}$

(a) (4 marks) Find A^{-1} .

(b) (4 marks) Solve the two following systems of equations:

$$-3y - 2z = -2$$

$$x - 4y - 2z = 1$$

$$-3x + 4y + z = 3$$

$$-3y - 2z = 1$$

$$x - 4y - 2z = 0$$

$$-3x + 4y + z = -2$$

5. Consider the functions:

$$f(x) = 2x + 1$$

$$g(x) = \frac{x}{x+1}$$

$$h(x) = \sqrt{2x+6}$$

(a) (3 marks) State the domain of f , g and h .

(b) (4 marks) Compute the following:

i. $f(-1+x)$

ii. $g(a+3)$

iii. $\frac{h(-2)}{1+h(x)}$

6. (a) (3 marks) If $\log x = 5$, $\log y = 2$, $\log z = 3$ find $\log\left(\frac{xy^3}{z}\right)$.

(b) (4 marks) In each case, express as a single logarithm with coefficient 1.

i. $5 \ln x - 3 \ln y - \ln 5$

ii. $\log_2 x - 8$

(c) (2 marks) Evaluate $\log_3\left(\frac{x}{5}\right) - \log_3\left(\frac{x}{15}\right)$

7. (10 marks) Solve the systems of equations corresponding to the augmented matrices below:

$$A = \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 0 \\ 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right], \quad B = \left[\begin{array}{ccc|c} 1 & 0 & -2 & 4 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right], \quad C = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 4 & -3 \\ 0 & 1 & 0 & 1 & 2 \end{array} \right]$$

8. In each of the following equations, solve for x . .

(a) (5 marks)

$$2^{2-3x} = 5^{2x-5}$$

(b) (5 marks)

$$\log_2(x) + \log_2(x - 3) = 2$$

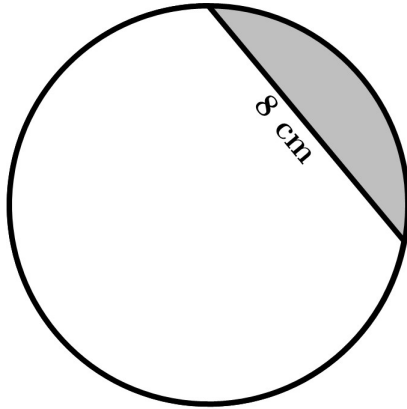
9. (4 marks) Find the **exact value** of

$$\cos \frac{5\pi}{4} + \tan \frac{4\pi}{3}$$

10. (7 marks) Solve for θ giving all solutions (in radians) contained in the interval $[0, 2\pi)$.

$$2 \sin \theta \cos \theta - \sqrt{3} \cos \theta = 0$$

11. (6 marks) Find the area of the shaded portion bounded by a chord of length 8cm in a circle of radius 3.5 cm (as illustrated below).



12. (6 marks) Given a system of three equations and two unknowns, explain the geometric significance of:
- one solution
 - no solutions
 - infinitely many solutions

Use drawings to illustrate your answers.

13. (4 marks) Find the **exact value** of $\cos \theta$ if $\tan \theta = -\frac{5}{4}$ and $\sin \theta > 0$.

14. (5 marks) Using the graph of $z = \log y$ with respect to x , illustrated below, find the formula for the function $y = f(x)$.

