

Dawson College Department of Mathematics

Instructors: N. Rossokhata, M. Ishii Sections: 1, 2

201-941-DW Engineering Math I
Final Exam
Fall 2025
Time Limit: 3 hours

Name: Solutions
Student ID: _____
Instructor's name _____

- Keep 4 decimal places for your calculations.
All lengths are in millimeters.
- Attempt EVERY question and show all your work. Please circle your final answers.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Only the calculators Sharp EL-531** are permitted.
- This exam booklet must be returned intact.

FORMULAS:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$S = R\theta_{\text{rad}} = R\theta_{\text{deg}} \left(\frac{\pi}{180} \right)$$

$$A = \frac{\theta_{\text{rad}} R^2}{2} = \frac{\theta_{\text{deg}} R^2}{2} \left(\frac{\pi}{180} \right)$$

Problem	Points	Score
1	5	
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	12	
11	8	
Total	100	

Bonus + 3 marks

1. (5 marks) Solve the linear system using Gauss-Jordan elimination:
- $$\begin{cases} -x + 4y - 2z = 1 \\ -2x + 10y = 6 \\ 3x - 11y + 10z = 9 \end{cases}$$

$$\left[\begin{array}{ccc|c} -1 & 4 & -2 & 1 \\ -2 & 10 & 0 & 6 \\ 3 & -11 & 10 & 9 \end{array} \right] R_1(-1)$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & -1 \\ -2 & 10 & 0 & 6 \\ 3 & -11 & 10 & 9 \end{array} \right] \begin{array}{l} R_2 + 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & -1 \\ 0 & 2 & 4 & 4 \\ 0 & 1 & 4 & 12 \end{array} \right] \begin{array}{l} \updownarrow \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & -1 \\ 0 & 1 & 4 & 12 \\ 0 & 2 & 4 & 4 \end{array} \right] \begin{array}{l} R_1 + 4R_2 \\ R_3 - 2R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 18 & 47 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & -4 & -20 \end{array} \right] R_3(-1/4)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 18 & 47 \\ 0 & 1 & 4 & 12 \\ 0 & 0 & 1 & 5 \end{array} \right] \begin{array}{l} R_1 - 18R_3 \\ R_2 - 4R_3 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -43 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

$$\begin{array}{l} x = -43 \\ y = -8 \\ z = 5 \end{array}$$

2. (5 marks) A company places an order for nuts, bolts and washers. The total number of all pieces is 2800. The number of nuts and bolts combined is 1800 more than the number of washers. The number of bolts is 3 times the number of washers. How many of each were ordered? (Set up the linear system and use Gauss-Jordan elimination to solve it.)

$x = \# \text{ nuts}$, $y = \# \text{ bolts}$, $z = \# \text{ washers}$

$$\begin{cases} x + y + z = 2800 \\ x + y = z + 1800 \\ y = 3z \end{cases} \rightarrow \begin{cases} x + y + z = 2800 \\ x + y - z = 1800 \\ y - 3z = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2800 \\ 1 & 1 & -1 & 1800 \\ 0 & 1 & -3 & 0 \end{array} \right] R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2800 \\ 0 & 0 & -2 & -1000 \\ 0 & 1 & -3 & 0 \end{array} \right] \begin{matrix} \uparrow \\ \downarrow \end{matrix}$$

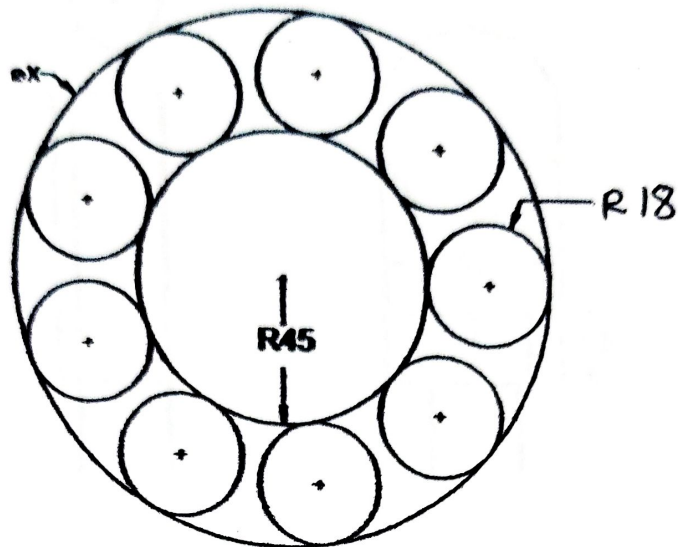
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2800 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -2 & -1000 \end{array} \right] \begin{matrix} R_1 - R_2 \\ R_3(-1/2) \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 2800 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 500 \end{array} \right] \begin{matrix} R_1 - 4R_3 \\ R_2 + 3R_3 \end{matrix}$$

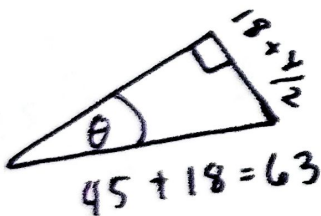
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 800 \\ 0 & 1 & 0 & 1500 \\ 0 & 0 & 1 & 500 \end{array} \right]$$

$x = 800 \text{ nuts}$
 $y = 1500 \text{ bolts}$
 $z = 500 \text{ washers}$

3. (10 marks) Find the diameter X and the clearance Y of the ballbearing :



$$\textcircled{X} = 2(45) + 4(18) = \textcircled{162 \text{ units}}$$



$$\theta = \frac{360^\circ}{18} = 20^\circ$$

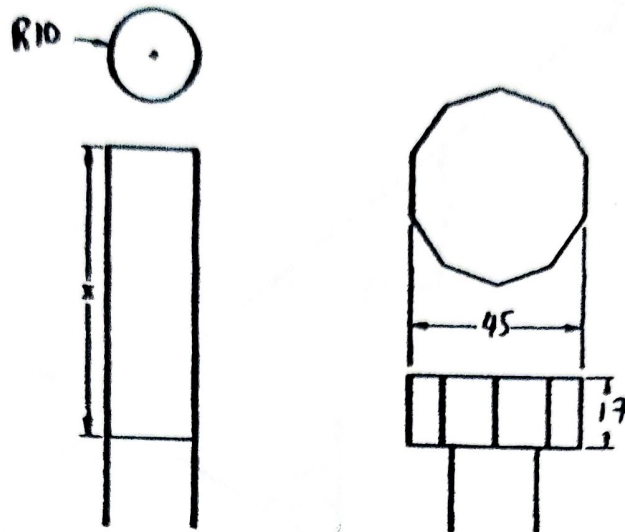
$$\sin(20^\circ) = \frac{18 + \frac{y}{2}}{63} \left(\frac{\frac{2}{2}}{\frac{2}{2}} \right) = \frac{36 + y}{126}$$

$$126 \sin(20^\circ) = 36 + y$$

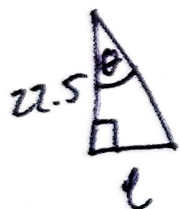
$$y = \textcircled{126 \sin(20^\circ) - 36}$$

$$\textcircled{\approx 7.0945 \text{ units}}$$

4. (10 marks) Find the length X of the piece of stock needed to stamp the bolt head.



$$V_2 = A_{\text{dec}} \cdot 17, \quad V_1 = \pi R^2 X = \pi 100 X, \quad V_1 = V_2$$



$$\theta = \frac{360}{20} = 18^\circ$$

$$\left\{ \begin{array}{l} \tan 18^\circ = \frac{l}{22.5} \\ l = 7.3107 \end{array} \right.$$

$$A_{\text{dec}} = 10 \left(\frac{2l}{2} \cdot 22.5 \right) = 1644.9075 \text{ units}^2$$

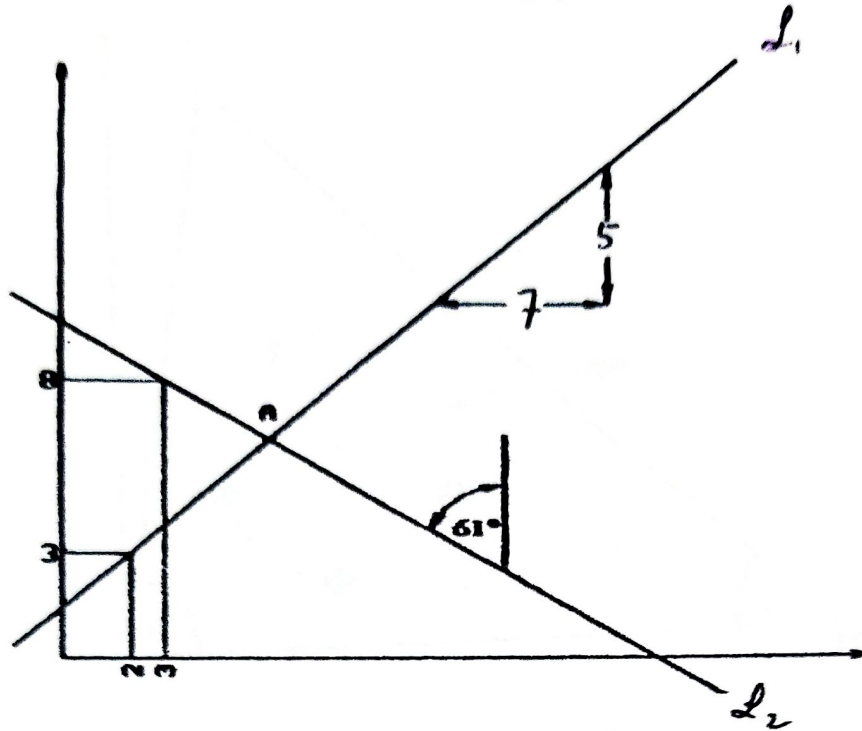
$$V_2 = A_{\text{dec}} \cdot 17 = 27963.4275$$

$$V_1 = \pi 100 X$$

$$V_1 = V_2 \rightarrow 100\pi X = 27963.4275$$

$$\left(X = 89.0104 \text{ units} \right)$$

5. (10 marks) Find the coordinates of point A in the graph



$$L_1: y = \frac{5}{7}x + b, (2, 3)$$

$$3 = \frac{10}{7} + b \rightarrow b = \frac{11}{7}$$

$$L_1: y = \frac{5}{7}x + \frac{11}{7}$$

$$L_2: m_2 = -\tan 29^\circ, (3, 8)$$

$$8 = (-\tan 29^\circ)3 + b$$

$$b = 8 + 3\tan 29^\circ$$

$$L_2: y = -0.5543x + 9.6629$$

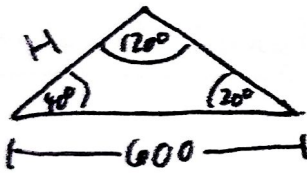
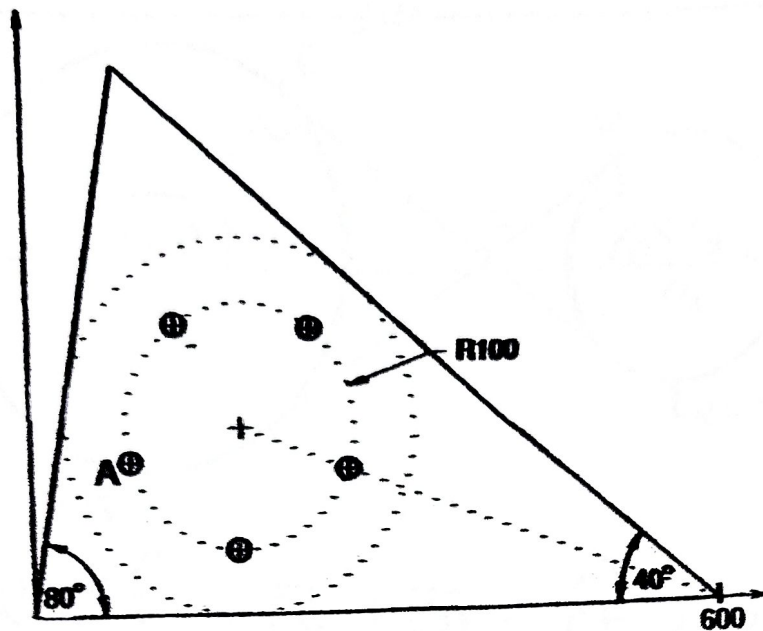
P.O.I. $0.7143x + 1.5714 = -0.5543x + 9.6629$

$$1.2686x = 8.0915$$

$$x = 6.3783$$

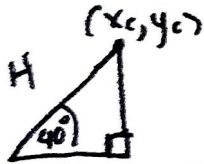
$$y = 6.1274$$

6. (10 marks) Find the coordinates of point A:



$$\frac{\sin 20^\circ}{H} = \frac{\sin 120^\circ}{600}$$

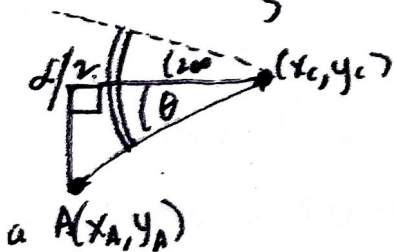
$$H \approx 236.9585$$



$$x_c = H \cos 40^\circ \approx 181.5207$$

$$y_c = H \sin 40^\circ \approx 152.3140$$

$$\alpha = \frac{360^\circ}{5} = 72^\circ$$



$$\theta = \frac{\alpha}{2} - 20^\circ = 16^\circ$$

$$x = 100 \cos 16^\circ \approx 96.1262$$

$$y = 100 \sin 16^\circ \approx 27.5637$$

point A

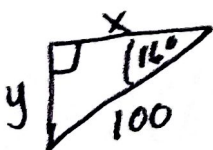
$$x_A = x_c - x$$

$$= 85.3945$$

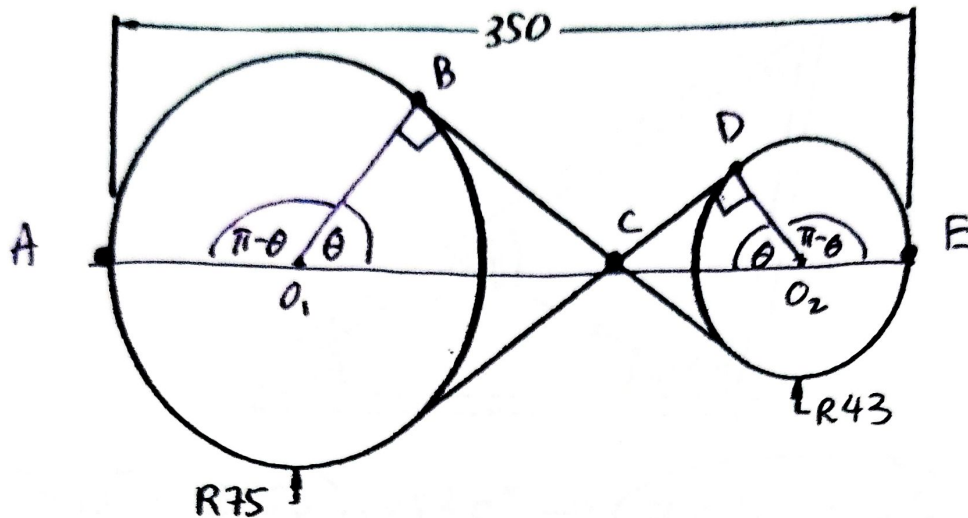
$$y_A = y_c - y$$

$$= 124.7503$$

$$(85.3945, 124.7503)$$



7. (10 marks) Find the beltlength:



$$\text{Belt length } L = (\widehat{AB} + \overline{BC} + \overline{CD} + \widehat{DE}) \cdot 2$$

similar triangles: $\overline{O_1C} = x$

$$\frac{75}{x} = \frac{43}{350 - 75 - 43 - x}$$

$$\frac{75}{x} = \frac{43}{232 - x}$$

$$x = 147.4576$$

$$\overline{O_2C} = 84.5424$$

$$\overline{BC} = \sqrt{x^2 - 75^2} \approx 126.9596 \quad \overline{CD} = \sqrt{(\overline{O_2C})^2 - 43^2} \approx 72.7902$$

Find θ (radians)

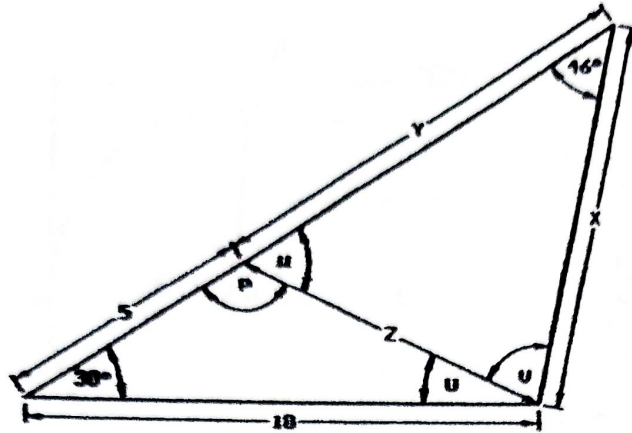
$$\cos \theta = \frac{75}{147.4576} \rightarrow \theta = \arccos\left(\frac{75}{147.4576}\right) \approx 1.0372 \text{ rad}$$

$$\text{arclength } s = R\theta \rightarrow \widehat{AB} = 75(\pi - \theta) \approx 157.8294$$

$$\widehat{DE} = 43(\pi - \theta) \approx 90.4889$$

$$L = 2(\widehat{AB} + \overline{BC} + \overline{CD} + \widehat{DE}) = \boxed{896.1362 \text{ units}}$$

8. (10 marks) Find the values of the letters X, Y, Z, U, V, W and P:



$$z^2 = 5^2 + 10^2 - 2(50)\cos 30^\circ \rightarrow z = 6.1966 \text{ units}$$

$$\frac{6.1966}{\sin 30^\circ} = \frac{5}{\sin u} \rightarrow u = 23.7938^\circ$$

$$p = 180 - 30 - 23.7938 = 126.2062^\circ = p$$

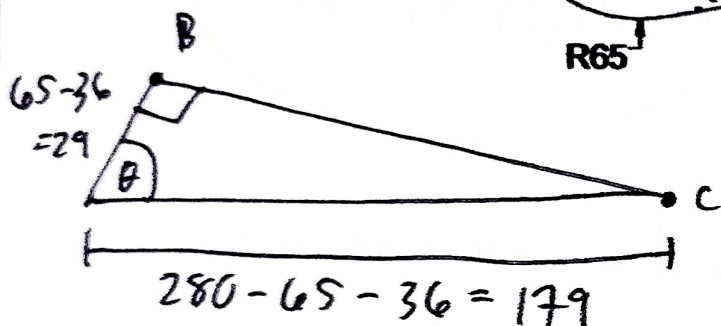
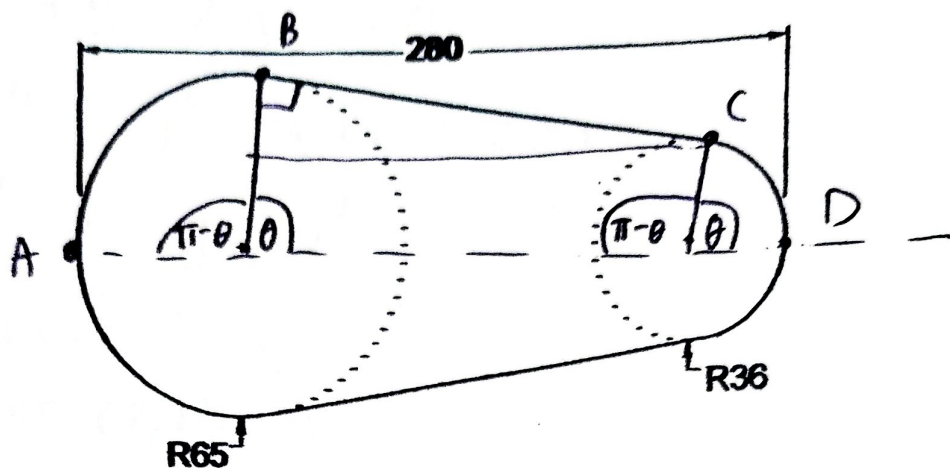
$$\omega = 180 - p = (53.7938^\circ = \omega$$

$$V = 180 - \omega - 46^\circ = 80.2062^\circ = V$$

$$\frac{6.1966}{\sin 46^\circ} = \frac{Y}{\sin 80.2062^\circ} \rightarrow Y = 8.4887 \text{ units}$$

$$\frac{6.1966}{\sin 46^\circ} = \frac{x}{\sin 53.7938} \rightarrow x = 6.9508 \text{ mits}$$

9. (10 marks) Find the AREA of the baseplate.



$$\begin{aligned} (\overline{BC})^2 &= 179^2 - 29^2 \\ \overline{BC} &= 176.6352 \end{aligned}$$

$$\cos \theta = \frac{29}{179} \rightarrow \theta = 1.4081 \text{ rad}$$

(2) marks

$$A_{SAB} = \frac{65^2 (\pi - \theta)}{2} = 3662.0032$$

$$A_{SCD} = \frac{36^2 (\theta)}{2} = 912.4488$$

$$A_{Trop} = \frac{(65 + 36) \overline{BC}}{2} = 8920.0776$$

$$\begin{aligned} \text{Area} &= 2(A_{SAB} + A_{SCD} + A_{Trop}) \\ &= 26989.0592 \text{ units}^2 \end{aligned}$$

10. (a) (4 marks) Find the radius and center of the circle with equation: $x^2 + y^2 - 4x - 2y - 20 = 0$

$$x^2 - 4x + y^2 - 2y = 20$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 20 + 5$$

$$(x-2)^2 + (y-1)^2 = 5^2$$

radius = 5

center (2, 1)

- (b) (4 marks) Solve for x: $\frac{x-1}{x^2-25} = \frac{2}{x-5} - \frac{x+2}{x+5}$

$$\left\{ \text{LCD } (x-5)(x+5) \right\}$$

$$\frac{(x-1)\cancel{(x-5)}\cancel{(x+5)}}{\cancel{(x-5)}\cancel{(x+5)}} = \frac{2\cancel{(x-5)}(x+5)}{\cancel{(x-5)}} - \frac{(x+2)(x-5)\cancel{(x+5)}}{\cancel{(x+5)}}$$

$$x-1 = 2(x+5) - (x^2-3x-10)$$

$$x-1 = 2x+10 - x^2 + 3x + 10$$

$$x^2 - 4x - 21 = 0$$

$$(x-7)(x+3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=7 & x=-3 \end{array}$$

$$x = -3 \text{ and } x = 7$$

(c) (4 marks) Solve for x : $2 + \sqrt{10-x} = \sqrt{3x-2}$

$$(2 + \sqrt{10-x})^2 = (\sqrt{3x-2})^2$$

$$4 + 4\sqrt{10-x} + (10-x) = 3x-2$$

$$4\sqrt{10-x} = 4x - 16$$

$$\sqrt{10-x} = x - 4$$

$$(\sqrt{10-x})^2 = (x-4)^2$$

$$10-x = x^2 - 8x + 16$$

$$0 = x^2 - 7x + 6$$

$$0 = (x-1)(x-6)$$

$$\downarrow$$
$$\cancel{x=1}$$

$$\downarrow$$
$$\boxed{x=6 \text{ only}}$$

11. Solve the following equations for x:

(a) (4 marks) $2^{x+1} = 5^x$

$$\ln(2^{x+1}) = \ln(5^x)$$

$$(x+1) \ln 2 = x \ln 5$$

$$x \ln 2 + \ln 2 = x \ln 5$$

$$x \ln 2 - x \ln 5 = -\ln 2$$

$$x(\ln 2 - \ln 5) = -\ln 2$$

$$x = \frac{-\ln 2}{\ln 2 - \ln 5} = \frac{\ln 2}{\ln 5 - \ln 2} \approx 0.7565$$

(b) (4 marks) $2\log_3(x) - \log_3(6x+7) = 0$

$$2\log_3 x = \log_3(6x+7)$$

$$\log_3 x^2 = \log_3(6x+7)$$

$$x^2 = 6x+7$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$\downarrow$
 $x = 7$
only

$\downarrow$
 ~~$x = -1$~~

BONUS QUESTION WORTH 3 EXTRA MARKSSolve the following equation for x : $2^{x+1} + 4^x = 80$

$$\begin{aligned} & \underline{2^{x+1} + 2^{2x} = 80} \\ & 2 \cdot 2^x + (2^x)^2 = 80 \quad \left\{ \begin{array}{l} y = 2^x \end{array} \right. \\ & y^2 + 2y = 80 \\ & \underline{y^2 + 2y - 80 = 0} \\ & (y - 8)(y + 10) = 0 \\ & \begin{array}{cc} \downarrow & \downarrow \\ y = 8 & \cancel{y = -10} \end{array} \quad \left\{ \begin{array}{l} y = 2^x > 0 \end{array} \right. \\ & 2^x = 8 \rightarrow \boxed{x = 3} \end{aligned}$$