Dawson College
Mathematics Department
FINAL EXAMINATION
Engineering Mathematics II
201-942-DW section 00001

December 22nd, 2023

Instructor: Oxana Cerba

Time: 9:30-12:30

Instructions:

- Print your name and student ID number in the space provided on the Cover Sheet.

- All questions are to be answered directly on the examination paper in the space provided. Show your complete work and give explanations.

- ONLY SHARP EL-531X, XG or XT are permitted.

This examination consists of 13 questions. Please ensure that you have a complete examination.

This examination must be returned intact.

Question 1. **[2+3+3 marks]** Evaluate each of the following limits.

(a) $\lim_{x \to 9} \frac{5}{2+\sqrt{x}}$

(b) $\lim_{x \to 5} \frac{x^2+2x-35}{x^2-x-20}$

(c) $\lim_{x \to \infty} \frac{2x^2+3x-5}{3x^3-2x+1}$

Question 2. [4 marks]

Given $f(x) = \frac{1}{4+3x}$. Using the limit definition of the derivative find $f'(x)$. (No points will be given for other method of solutions.)

Question 3. **[15 marks]** Find the derivative of each function below. DO NOT SIM-
PLIFY

(a) $y = (7x^3 - \sqrt{x})^{14}$

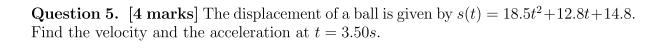(b) $y = \frac{1+x^2}{\sqrt{x+1}}$

(c) $y = (x^2 - 1)^5 (x - 3)^4$

(d) $y = \frac{\cot(x)}{x^2}$

(e) $y = \sqrt{x} arccsc(x)$

Question 4. [5+5 marks] Using implicit differentiation find y'.

(a) $\sqrt{xy} = 2x + y^2$

(b) $xy^2 = x\cos(y) + y\sin(x)$

Question 5. [**4 marks**] The displacement of a ball is given by $s(t) = 18.5t^2 + 12.8t + 14.8$. Find the velocity and the acceleration at $t = 3.50s$.

Question 6. [**6 marks**] Find the equations of the tangent and normal lines to the curve $y = x\cos(x) + 4$ at the point $(0, 4)$.

Question 7. [**8 marks**] A car leaves an intersection traveling west. Its position 4 sec later is 20ft from the intersection. At the same time another car leaves the same intersection heading north so that its position 4sec later is 28ft from the intersection. If the speed of the cars at that instant of time is 9 ft/sec and 11 ft/sec respectively, find the rate at which the distance between the two cars is changing.

Question 8. **[5+5 marks]** Given the function $f(x) = x^4 - 2x^2 + 4$

(a) Find max/min and intervals where the function is increasing /decreasing.

(b) Find inflection points and intervals of concavity.

Question 9. **[8 marks]** A metal box with a square base and vertical sides is to contain $1024cm^3$. The material for the top and bottom costs $5c/cm^2$ and the material for the sides costs $2.5c/cm^2$. Find the least cost of the box.

Question 10. [**4 marks**] The blade of a reciprocating saw has a displacement (in m) function given by $y = 2.5\sin(8\pi t) - 1.5\cos(4\pi t)$. Find the velocity of the blade at 2.0s.

Question 11. [**8 marks**] An airplane is flying horizontally at an altitude of 5 km and with speed of 600 km/h. It passes directly over an observer on the ground. Determine how fast the angle between the airplane and the observer is increasing, in rads/h, 1 minute after the airplane passed the observer.

Question 12. [3+3+3 marks]

(a) $\int x^7(1+x^8)^3 1\,dx$

(b) $\int \frac{1+x^2}{\sqrt{x}}\,dx$

(c) $\int \frac{dx}{1+5x}$

Question 13. [6 marks]

Find the equation of the curve that passes through the point $(4, 1)$, has slope 48.25 at that point, and has the second derivative $y" = -\frac{1}{4}x^{-\frac{3}{2}} + 6x$.