DAWSON COLLEGE
MATHEMATICS DEPARTMENT

Final Examination
Fall 2024

Engineering Mathematics II
201-942-DW section 01

Date: Thursday, December 12th, 2024 at 9:30am
Instructor: C. Farnesi

1. Evaluate the following limits, or show that they do not exist:

   a) **[3 Pts]**

   $$\lim_{x\to -3} \frac{x^3 + 3x^2 - 5x - 15}{x^2 - 9}$$

   b) **[1 Pt]**

   $$\lim_{x\to 2} \frac{x^2 + 4}{x - 2}$$

   c) **[3 Pts]**

   $$\lim_{x\to \infty} \frac{20x^2 - 15x + 7}{13 + 9x - 44x^2}$$

2. Consider the function:

   $$f(x) = x^3 - 2x + 9$$

   a) **[8 Pts]** Use **only** the limit definition of the derivative (the delta-method / four-step process) to find the derivative of $f(x)$ given above.

   b) **[1 Pt]** Verify your answer in part (a) using derivative rules.

   c) **[4 Pts]** Find the equation of the normal line to the function $f(x)$ at the point $(-2,5)$.

3. **[3 Pts]** A rocket is launched into space. Its height in kilometers after t hours is given by the function:

$$f(t) = 0.1t^3 - 20t^2 + 50{,}000t$$

How fast is the rocket's height changing after 10 hours?

4. **[12 Pts]** Find the derivatives of the following functions. Do NOT simplify your answer.

   a)
   $$y = \frac{13}{\sqrt[3]{3x - \cot x^3}}$$

   b)
   $$y = \tan(x^7 + 7) - \sec 7x + \operatorname{arccot} 7x^7$$

   c)
   $$y = \csc x^2 \cdot \arcsin 2x$$

   d)
   $$y = \frac{(9x + 2)^9}{\sqrt{2x - 9}}$$

5. **[3 Pts]** For what value(s) of x does the following function have a horizontal tangent line?
   $$y = (3x^2 - 7)^{37}$$

6. Consider the implicitly defined function:
   $$x^3y^3 - 3x = 3y^2 - 1$$

   a) **[6 Pts]** Find the derivative y'.

   b) **[1 Pt]** Calculate the slope of the tangent line at the point $(2,1)$.

7. **[4 Pts]** Find $f''(0)$ if:
   $$f(x) = x^3 + \sin 5x$$

8. Consider the function:
   $$f(x) = 2x^3 + 3x^2 - 12x + 1$$

   a) **[6 Pts]** Using the first derivative, determine the intervals where $f(x)$ is increasing, where it is decreasing, and identify any relative extrema.

b) **[6 Pts]** Determine the intervals where $f(x)$ is concave up, where it is concave down, and identify any inflection points.

c) **[2 Pts]** Use the second derivative to verify your results from part (a).

9. **[6 Pts]** A parent is standing 6 feet away from their child in a sled. The parent is holding the sled by a rope, 2 feet above the point of attachment to the sled. Without moving themselves, the parent pulls the rope of the sled **towards them** at a rate of 0.3 ft/sec. How fast is the horizontal distance between the sled and the parent changing?

10. **[10 Pts]** Starting with a piece of cardboard that is 24" by 36", you cut out equal squares from all four corners and fold up the resulting flaps to create an open box. Find the dimensions of the largest box possible. (Remember to verify that these dimensions indeed give a maximum.)

11. The velocity (in cm/sec) of an object moving on a straight line is given by:

$$v = 1.8t^2 - 0.6t + 0.09$$

a) **[1 Pt]** What is the object's initial velocity?

b) **[1 Pt]** Find an equation for the object's acceleration.

c) **[2 Pts]** When will the object have an acceleration of 17.4 cm/sec^2?

d) **[2 Pts]** Find an equation for the object's displacement if its initial position is 0.

e) **[1 Pt]** What will be the object's position after 10 seconds?

12. Find the following antiderivatives and simplify your answers:

a) **[3 Pts]**

$$\int \frac{9x^2 + 7x - 3}{\sqrt{x}}\, dx$$

b) **[6 Pts]**

$$\int (5 + 9x^2)\,(15x + 9x^3)^6\, dx$$

13. **[5 Pts]** Find the function $f(x)$ that has an instantaneous rate of change of 14.5 at the point $(2,-8)$ and such that its second derivative is:

$$f''(x) = \frac{3}{4}x - 14$$

Answers

1. a) $\frac{-2}{3}$ b) DNE c) $\frac{-5}{11}$
2. a) b) $f'(x) = 3x^2 - 2$ c) $y = \frac{-1}{10}x + \frac{24}{5}$
3. 49 630 km/h
4. a) $y' = 13\left(\frac{-1}{3}\right)(3x - \cot x^3)^{-4/3}\left(3 + (\sec^2 x^3)(3x^2)\right)$

   b) $y' = \sec^2(x^7 + 7) \cdot (7x^6) - 7\sec 7x \tan 7x - \frac{49x^6}{1+(7x^7)^2}$

   c) $y' = (-\csc x^2 \cot x^2)(2x)\arcsin 2x + \frac{2}{\sqrt{1-(2x)^2}}\csc x^2$

   d) $y' = \frac{9(9x+2)^8(9)\sqrt{2x-9} - \frac{1}{2}(2x-9)^{-1/2}(2)(9x+2)^9}{\left(\sqrt{2x-9}\right)^2}$
5. $x = 0, x = \pm\sqrt{\frac{7}{3}}$
6. a) $y' = \frac{3-3x^2y^3}{3y^2x^3-6y}$ b) -0.5
7. $f''(0) = -119$
8. a) increasing on $(-\infty,-2) \cup (1,\infty)$ and decreasing on $(-2,1)$ with relative maximum at $(-2,21)$ and relative minimum at $(1,-6)$

   b) concave down on $(-\infty,-0.5)$ and concave up on $(-0.5,\infty)$ with inflection point at $(-0.5,7.5)$

   c) $f''(-2) < 0$ and $f''(1) > 0$ confirms results in part a
9. -0.316 ft/sec
10. The dimensions are approximately 4.71" by 14.58" by 26.58"

11. a) 0.09 cm/s b) $a = 3.6t - 0.6$ c) 5 sec d) $d = 0.6t^3 - 0.3t^2 + 0.09t$

e) 570.9 cm

12. a) $\frac{18}{5}x^{5/2} + \frac{14}{3}x^{3/2} - 6\sqrt{x} + C$ b) $\frac{1}{21}(15x + 9x^3)^7 + C$

13. $f(x) = \frac{x^3}{8} - 7x^2 + 41x - 63$