

Dawson College
Mathematics Department

Final Examination

201-MA1-DW Calculus-I Social Science

Fall – 2025

Student Name: _____

Student I.D. #: _____

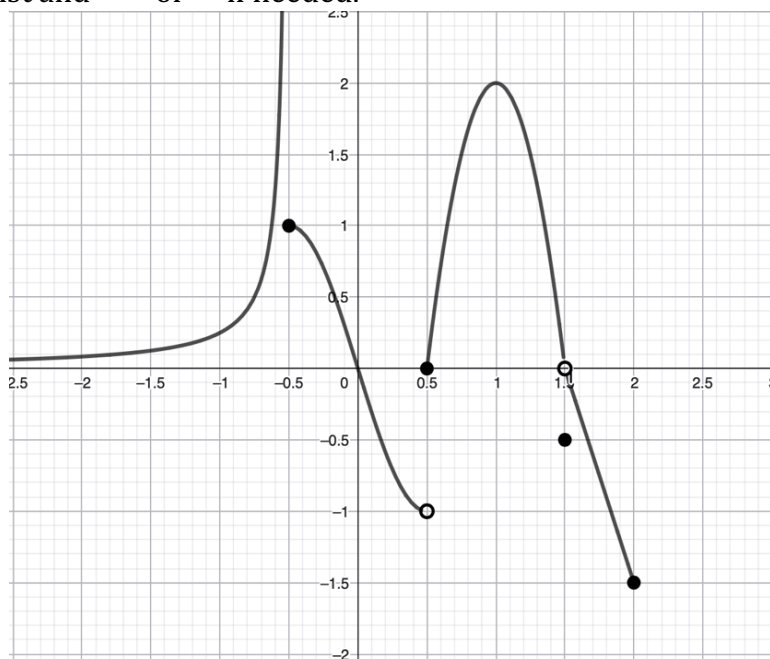
Instructor Name: _____

Instructors: G. Bobos, G. Chu, I. Gorelyshev, M. Moodi, I. Rajput, O. Veres

INSTRUCTIONS :

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer, use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 16 questions on 12 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

1. The graph of $y = f(x)$ is given below. Answer the following questions. Use DNE if the limit does not exist and $-\infty$ or ∞ if needed.



[2.5 marks] Find the following limits.

- a. $\lim_{x \rightarrow -\infty} f(x) =$ b. $\lim_{x \rightarrow -0.5^-} f(x) =$ c. $f(-0.5) =$
- c. $\lim_{x \rightarrow 0.5} f(x) =$ d. $\lim_{x \rightarrow 1.5} f(x) =$

e. [1.5 marks] Find the x value(s) where the function is discontinuous.

f. [1mark] State the condition for continuity that is violated at $x = 1.5$.

g. [1 mark] Find the absolute (global) extrema, if any.

h. [1 mark] Find the interval(s) where the derivative of f is positive.

2. Use algebraic techniques to evaluate the following limits, if they exist; show your work and write $-\infty$ or ∞ if needed.

a. [4 marks] $\lim_{x \rightarrow 2^+} \frac{\sqrt{x^2+21}-5}{x-2}$

b. [4 marks] $\lim_{x \rightarrow -1} \frac{\frac{5x}{2x+3}+5}{x^2+9x+8}$

3. Consider the following function:

$$f(x) = \frac{11x(x^2 + x)}{x^4 - x^2}$$

a. [3 marks] Find the equation of the horizontal asymptote(s) of $f(x)$.

b. [3 marks] Find the equation of the vertical asymptote(s) of $f(x)$.

4. [5 marks] The function that gives the cost of a certain commodity is defined by

$$C(x) = \begin{cases} \frac{x^2 + x - 12}{x - 3}, & 0 < x < 3 \\ x^2 - 2, & 3 \leq x < 10 \\ 2x + 5, & 10 \leq x < 20 \end{cases}$$

where x is the number of pounds of a certain commodity sold and $C(x)$ is measured in dollars. State the condition of continuity and determine the value(s) of x for which the function C is discontinuous.

5. The quarterly profit (in millions of dollars) of a company is given by

$$P(x) = 3x(x + 20) \quad (0 \leq x \leq 50)$$

where x (in millions of dollars) is the amount of money spent on salary per quarter.

a. [5 marks] Use the limit definition of derivative to find $P'(x)$.

b. [1 mark] What is the rate of change of the company's quarterly profit if the amount it spends on salary is \$ 20,000 000/quarter ($x = 20$)?

6.

- a. [3 marks] Differentiate and do not simplify the answer.

$$f(x) = \sqrt[4]{x}(x^3 + 5)^7 + x^e$$

- b. [3 marks] The number of subscribers to a new streaming platform is modeled by the function:

$$S(t) = (80 + 3t)^{\frac{2}{3}} \quad (1 \leq t \leq 30)$$

where $S(t)$ in millions, represents the number of subscribers at the end of week t .

Find the rate of change of the number of subscribers at the end of week 15.

7. [5 marks] Find the slope of the tangent line to the graph of the function

$$f(x) = \frac{\sin 3x}{\cos^2 x} + \pi^3 \quad \text{at } x = \pi$$

8. A market study estimates the number of active users (in millions) to be

$$U(t) = e^{0.02t^2 - 0.12t + 4.3}, \quad 0 \leq t \leq 4,$$

where t represents the number of decades since 2010 ($t = 0$ corresponds to the year 2010).

- a. [1 mark] What is the projected number of active users in 2030?

- b. [3 marks] How fast is the number of active users changing in 2030?

- c. [2 marks] Find $U''(2)$.

9. [5 marks] The demand function for the Zenith e-Bike is

$$p = 180 - 0.02x \ln x$$

where x denotes the number of e-bikes sold and p is the price per bike in hundreds of dollars.

- a. Write the revenue function $R(x)$.

- b. Find the rate of change of revenue when $x = 400$.

10. [6 marks] Use logarithmic differentiation to find the derivative of

$$y = (x^2 + 3)^{\tan x}$$

11. Given the price-demand equation $p = 1000 - 0.05x$.

a. [2 marks] Find the exact revenue generated by the item number 3001.

b. [3 marks] Estimate this value using the marginal revenue function.

12. [6 marks] Find an equation of the tangent line to the graph of the function given by $x^2 + 5xy + y^3 = 15$ at the point (2,1).

13. [6 marks] The demand function for a certain make of ink-jet cartridge is

$$20x^2 + 3p^2 = 2192$$

where p is the unit price in dollars and x is the quantity demanded each week, measured in units of a thousand. How fast is the quantity demanded increasing when the unit price per cartridge is \$8 and the price is dropping at the rate of \$0.2/cartridge/week?

14. [5 marks] The daily total cost (in dollars) incurred by Algae-Bruh for producing x cartons of premium roasted seaweed snacks is

$$C(x) = 0.001x^3 + 20x + 2000, \quad 0 \leq x \leq 300.$$

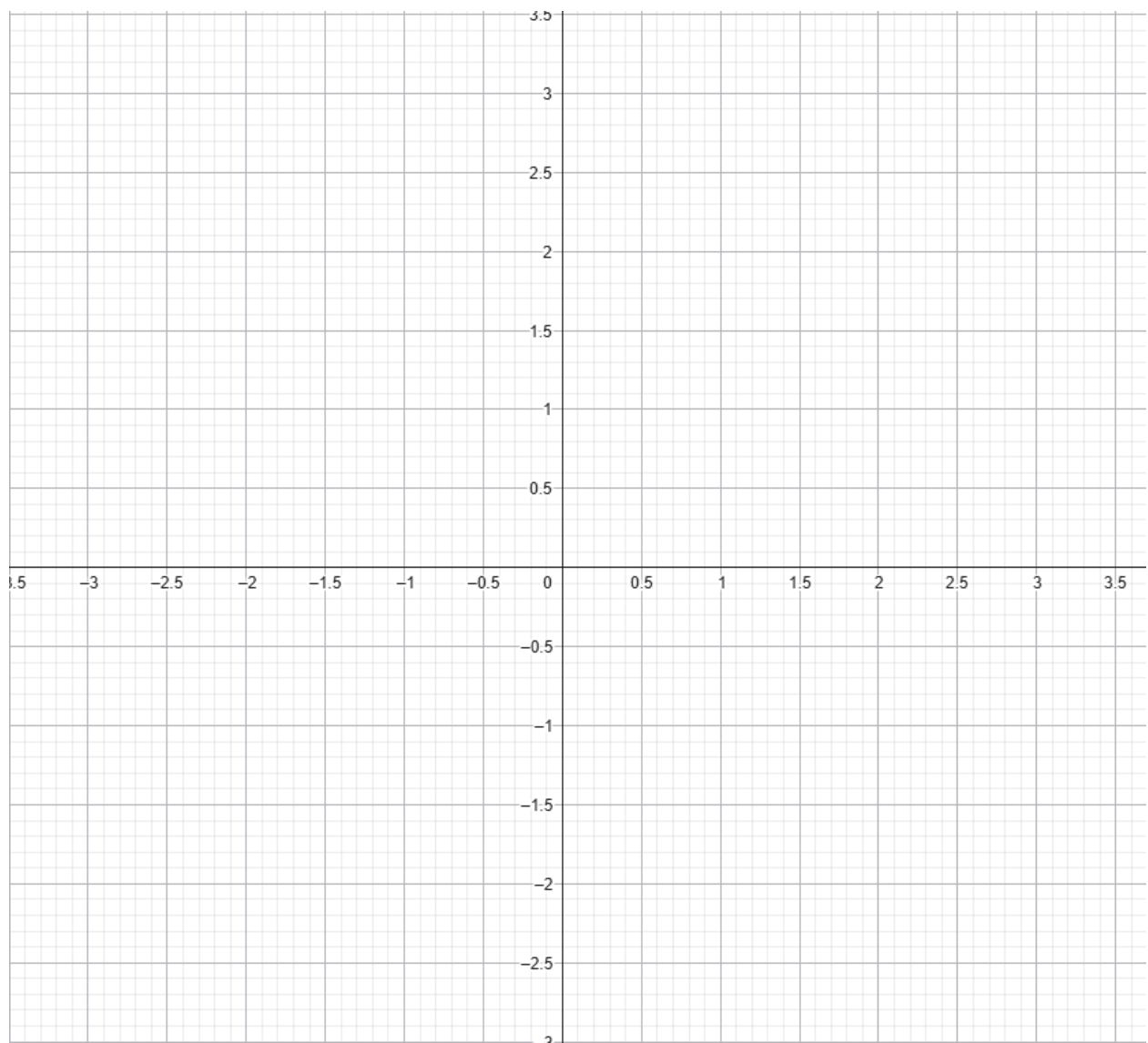
What level of daily production will minimize the average cost?

15. [6 marks] A box with an open top is to be constructed from a square piece of cardboard, 36 inches wide, by cutting out a square from each of the four corners and folding up the sides. Find the largest possible volume of such a box.

16. The path of a moving object is given by the following function:

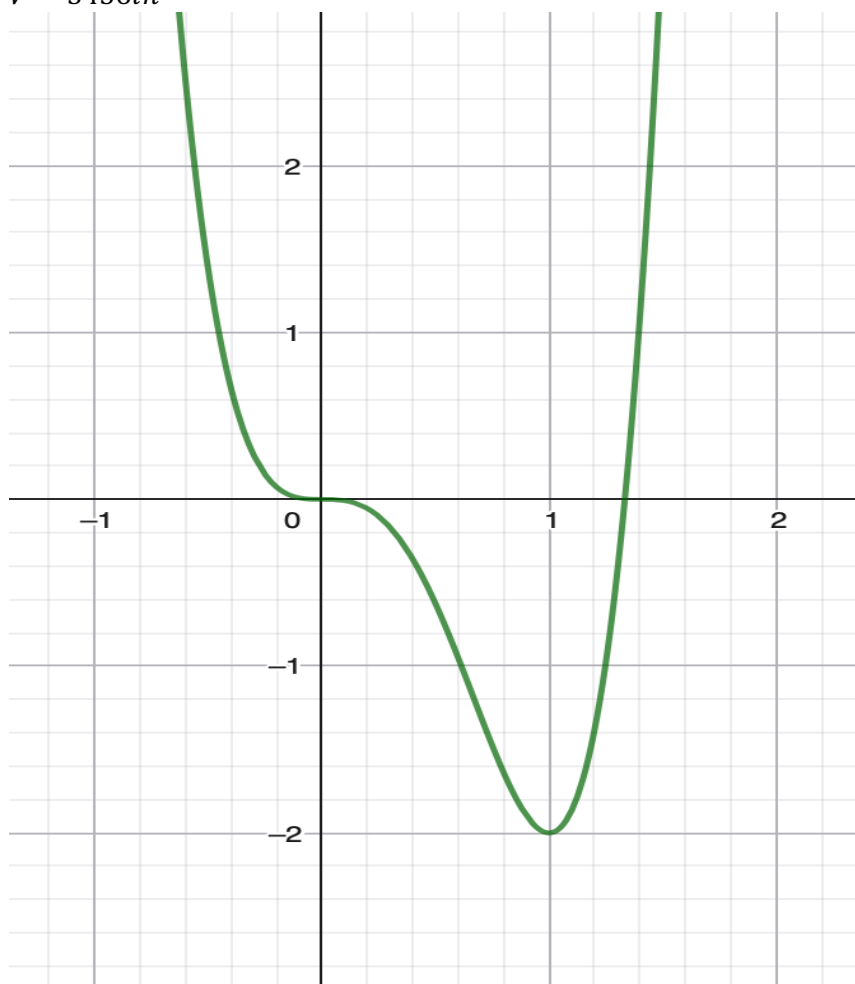
$$f(x) = 6x^4 - 8x^3$$

- a. [3 marks] Find the x -intercept(s) and y -intercept.
- b. [2 marks] Find the intervals where f increases and decreases.
- c. [1 mark] Find the relative extrema, if any.
- d. [2 marks] Find the intervals where the graph of f is concave upward and concave downward (convex, concave).
- e. [1 mark] Find the inflection point(s), if any.
- f. [3 marks] Label all these important points and sketch the graph of f .



ANSWERS:

1. a. 0; b. inf; c. 1; d. dne, e. 0; f. -0.5, 0.5, 1.5; g. 3rd condition is not satisfied. h. No abs max. f(2)=-1.5 is absolute minimum; i. $(-\infty, -0.5) \cup (0.5, 1)$
2. a. 2/5; b. 15/7;
3. a. y=0; b. x=1
4. x=10
5. a. 6x+60; b. 180 mill\$/quarter
6. a. $\frac{1}{4}x^{-\frac{3}{4}}(x^3 + 5)^7 + 21\sqrt[4]{x}x^2(x^3 + 5)^6 + ex^{e-1}$ b. 0.4 mill/week
7. -3
8. a. 62.8 mill; b. -2.5 mill. users/decade; c. 2.6 mill. users/decade²
9. a. $R(x) = 180x - 0.02x^2 \ln x$; b. 76.1 hundreds of \$/e-bike sold.
10. $(x^2 + 3)^{\tan x} [\sec^2 x \ln(x^2 + 3) + \frac{2x \tan x}{x^2 + 3}]$
11. a. 699.95; b. 700
12. $y = -\frac{9}{13}x + \frac{31}{13}$
13. The quantity is increasing at a rate of 24 cartidges /week
14. x=100
15. $V = 3456\text{in}^3$



16.