Sequences

Definition: A **sequence** is a set of numbers that is ordered.

For example:

- The days of the month: 1, 2, 3, …
- Multiples of 3: 3, 6, 9, …
- Successive powers of 10: $10^1, 10^2, 10^3, \ldots$
- Squares of integers: 1, 4, 9, 16, …

- Notice that in each set, there is a first number, a second number, a third number, and so forth.
- The successive numbers are called the **terms of the sequence**.

Definition: A **finite** sequence has a first term and a last term, whereas an **infinite** sequence has a first term but no last term.

For example:

- $\{1, 3, 5, 7, 9, 11\}$ is a finite sequence
- $\{\pi, \pi^2, \pi^3, \ldots, \pi^n, \ldots\}$ is an infinite sequence

- Mathematically, we think of a sequence as a function.

Definition: An **infinite sequence** $\{a_n\}$ is a function whose domain is the set of natural numbers and whose range is the set of term values.

For example, consider the infinite sequence {1, 3, 5, 7, 9, 11, …}.

- Term 1 has value of 1, term 2 has value of 3, term 3 has value of 5, and so forth.
- The domain is $D\colon\{1,2,3,4,\ldots\}$ and the term values are the range $R\colon\{1,3,5,7,\ldots\}$.

- Note that instead of using function notation $f(n)$ for a sequence, we write:
  $\{a_n\} = \{a_1, a_2, a_3, a_4, \ldots, a_n, \ldots\}$, where a_n is called the nth term of the sequence.
  For example:
    - The terms of the infinite sequence with general term $a_n = \frac{1}{n}$ would look like: $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots\right\}$

<u>Ex. 1</u>: Find the first four terms of a sequence whose nth term is $a_n = 3 + (-1)^n$

$$a_1 = 2, a_2 = 4, a_3 = 2, a_4 = 4$$

<u>Ex. 2</u>: Find the first four terms of a sequence whose nth term is $a_n = \frac{2n}{1+n}$

$$a_1 = 1, a_2 = \frac{4}{3}, a_3 = \frac{3}{2}, a_4 = \frac{8}{5}$$

<u>Ex. 3</u>: Find the first three terms of the sequence given by $a_n = \frac{(-1)^n}{2n-1}$

$$a_1 = -1, a_2 = \frac{1}{3}, a_3 = \frac{-1}{5}, a_4 = \frac{1}{7}$$

<u>Ex. 4</u>: Find the 37th term of the sequence $a_n = \frac{(-1)^n}{2n-1}$

$$a_{37} = \frac{-1}{73}$$

<u>Ex. 5</u>: Given the infinite sequence {3,5,7,9,11,13,15…}

a) Discover a pattern. <u>Ans</u>: $a_n = a_{n-1} + 2$

b) Write 3 more terms of the sequence. <u>Ans</u>: 17,19,21

<u>Ex. 6</u>: Given the infinite sequence {1,5,9,13,…}

a) Discover a pattern. <u>Ans</u>: $a_n = a_{n-1} + 4$

b) Write 4 more terms of the sequence. <u>Ans</u>: 17,21,25,29

There exist two types of sequences that are of special interest mathematically because they model real world situations, and because their formulas are easy to deduce. They are **arithmetic** sequences and **geometric** sequences.

Definition: An **arithmetic sequence** is a sequence in which successive terms increase or decrease by a constant amount d, called the **common difference**, such that:

$$d = a_n - a_{n-1}$$

For example:

- {3,10,17,24,31…} is an arithmetic sequence with common difference $d = 7$.
- {18,16,14,12,10,…} is an arithmetic sequence with common difference $d = -2$.

Definition: A **geometric sequence** is a sequence in which successive terms increase or decrease by a constant factor r, called the **common ratio**, such that:

$$r = \frac{a_n}{a_{n-1}}$$

For example:

- {3,6,12,24,48,…} is a geometric sequence with common ratio $r = 2$.
- {128,64,32,16,8,…} is a geometric sequence with common ratio $r = \frac{1}{2}$.

Ex. 7: Determine if the following infinite sequences are arithmetic, geometric, or neither.

a) $\{4,7,10, \dots\}$ Ans: Arithmetic because we have $d = 3$.

b) $\{1,3,9,27,81, \dots\}$ Ans: Geometric because we have $r = 3$.

c) $\{64, -32, 16, -8, 4, \dots\}$ Ans: Geometric because we have $r = -\frac{1}{2}$.

d) $\{2,6,24, \dots\}$ Ans: Neither.

- It is possible to develop formulas for the nth term of both arithmetic and geometric sequences.

Arithmetic Sequences

Consider the infinite sequence $\{3, 10, 17, 24, 31, \ldots\}$.

Notice that the first term $a_1 = 3$ and the common difference is $d = 7$. So:

$a_1 = 3$	
$a_2 = 3 + 7$	$= 3 + 1(7)$
$a_3 = 3 + 7 + 7$	$= 3 + 2(7)$
$a_4 = 3 + 7 + 7 + 7$	$= 3 + 3(7)$
$a_5 = 3 + 7 + 7 + 7 + 7$	$= 3 + 4(7)$
…	
$a_n = 3 + (n - 1)7$	

Definition: For an arithmetic sequence with first term a_1 and common difference d, the nth term of the sequence is given by the formula:

$$a_n = a_1 + (n - 1)d$$

Ex. 8: Write the formula for the nth term of the sequence $\{3, 9, 15, 21, \ldots\}$.

We have $a_1 = 3$ and $d = 6$, so $a_n = 3 + (n - 1)6 = 3 + 6n - 6 = 6n - 3$.

Ex. 9: Consider the sequence $\{17, 22, 27, 32, \ldots\}$. Find the formula for the nth term of the sequence and use it to find the value of the 100th term (ie: term a_{100}).

We have $a_1 = 17$ and $d = 5$, so $a_n = 17 + (n - 1)5$ and $a_{100} = 17 + (99)5 = 512$

Ex. 10: Find the number of terms in the finite arithmetic sequence: $\{9, 2, -5, \ldots, -40\}$.

We have $a_1 = 9$, $d = -7$ and we set $a_n = -40$. So using $a_n = a_1 + (n - 1)d$ we have $-40 = 9 + (n - 1)(-7)$ and find $n = 8$.

Note that we can check this by letting $n = 8$: $a_8 = 9 + (7)(-7) = -40$.

Geometric Sequences

Consider the sequence $\{3,6,12,24,48,\ldots\}$.

Notice that the first term $a_1 = 3$ and the common ratio is $r = 2$. So:

$a_1 = 3$	
$a_2 = 3 \cdot 2$	$= 3 \cdot 2$
$a_3 = 3 \cdot 2 \cdot 2$	$= 3 \cdot 2^2$
$a_4 = 3 \cdot 2 \cdot 2 \cdot 2$	$= 3 \cdot 2^3$
$a_5 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$= 3 \cdot 2^4$
…	
$a_n = 3 \cdot 2^{n-1}$	

Definition: For a geometric sequence with first term $a_1 = a$ and common ratio r, the nth term of the sequence is given by the formula:

$$a_n = a \cdot r^{n-1}$$

Ex. 11: Write the formula for the nth term of the sequence $\left\{45, -15, 5, -\frac{5}{3}, \ldots\right\}$.

We have $a = 45$ and $r = -\frac{1}{3}$, so $a_n = 45 \cdot \left(-\frac{1}{3}\right)^{n-1} = 45\left(-\frac{1}{3}\right)^n\left(-\frac{1}{3}\right) = -15\left(-\frac{1}{3}\right)^n$.

Ex. 12: Consider a geometric sequence with $a = 35$ and $r = 1.05$. Find the formula for the nth term of the sequence and use it to find the value of the 100th term.

We have $a_n = 35 \cdot (1.05)^{n-1}$ and so $a_{100} = 35 \cdot (1.05)^{99} \approx 4383.375$

- In some cases, we may want to know how a sequence behaves in the long run.

Convergence of sequences

Definition: A sequence a_n **converges** and has the limit L, such that:

$$\lim_{n\to\infty} a_n = L$$

if the terms of the sequence can be made as close as possible to L by taking n sufficiently large.

** If a sequence is not convergent, it is **divergent**.*

Ex. 13: Consider the sequence $\left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \dots\right\}$. We can deduce that as n increases, the terms of the sequence get smaller. But will they decrease forever?

Taking the limit, we have $\lim\limits_{n\to\infty} \frac{1}{n} = 0$. Therefore, the sequence converges.

Ex. 14: Consider the sequence $\{\sqrt{n}\} = \{1, \sqrt{2}, \sqrt{3}, \sqrt{4} \dots\}$. We can deduce that as n increases, the terms of the sequence get bigger. But will they increase forever?

Taking the limit, we have $\lim\limits_{n\to\infty} \sqrt{n} = \infty$. Therefore, the sequence diverges.

Ex. 15: Does the geometric sequence $\{a_n\} = \{3 \cdot 2^{n-1}\}$ converge or diverge?

Taking the limit, we have $\lim\limits_{n\to\infty} 3 \cdot 2^{n-1} = \infty$. Therefore, the sequence diverges.

Ex. 16: Does the geometric sequence $\{a_n\} = \left\{15\left(\frac{1}{3}\right)^n\right\}$ converge or diverge?

Taking the limit, we have $\lim\limits_{n\to\infty} 15\left(\frac{1}{3}\right)^n = 0$. Therefore, the sequence converges.

Ex. 17: Does the geometric sequence $\{a_n\} = \left\{15\left(-\frac{1}{3}\right)^n\right\}$ converge or diverge?

Taking the limit, we have $\lim\limits_{n\to\infty} 15\left(-\frac{1}{3}\right)^n = 0$ also. Therefore, the sequence converges.

Ex. 18: Does the geometric sequence $\{a_n\} = \{3(-2)^n\}$ converge or diverge?

Taking the limit, we have $\lim_{n\to\infty} 3(-2)^n = DNE$. Therefore, the sequence diverges.

Ex. 19: Does the arithmetic sequence $\{a_n\} = \{6n - 3\}$ converge or diverge?

Taking the limit, we have $\lim_{n\to\infty}(6n - 1) = \infty$. Therefore, the sequence diverges.

Ex. 20: Consider the arithmetic sequence with $a_1 = 4$ and $d = -3$. Does the sequence converge?

Here, $a_n = 4 + (n-1)(-3) = 4 + 3 - 3n = 7 - 3n$. Taking the limit, we have $\lim_{n\to\infty}(7 - 3n) = -\infty$. Therefore, the sequence diverges.

Ex. 21: Consider the sequence $\{a_n\} = \{c, c, c, \dots\}$, where c is a constant.

a) Show that this is a arithmetic sequence and write a formula for the general term a_n.

   We have a first term $a_1 = c$ and a common difference $d = 0$, such that $a_n = a_1 + (n-1)d = c + (n-1)(0) = c$.

b) Show that this is also an geometric sequence and write a formula for the general term a_n.

   We have a first term $a = c$ and a common ratio $r = 1$, such that $a_n = ar^{n-1} = c(1)^{n-1} = c$.

c) Show that the sequence converges.

   We take the limit: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c = c$, therefore the sequence converges.

- It is possible to find the finite sum of both arithmetic and geometric sequences.

Sums of sequences

Before we begin, we need to introduce a new notation.

Suppose we want to add up a list of numbers, for example, the first 5 terms of a sequence. One way we can do it is by using a summation notation:

$$a_1 + a_2 + a_3 + a_4 + a_5 = \sum_{k=1}^{5} a_k$$

Ex. 22: Find:

a) $\sum_{k=1}^{5} k$

$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15$

Notice that this is the sum of the first 5 terms of the arithmetic sequence with $a_1 = 1$ and $d = 1$.

b) $\sum_{k=1}^{5} 2$

$\sum_{k=1}^{5} 2 = 2 + 2 + 2 + 2 + 2 = 10$

Notice that this is the sum of the first 5 terms of the geometric sequence with $a = 2$ and $r = 1$.

Definition: A **partial sum S_n** represents the sum of the first n consecutive terms of a sequence.

Ex. 23: Find the partial sum S_6 for the arithmetic sequence $\{a_n\} = \{5n + 1\}$.

Here, the first 6 terms of the sequence are $\{6,11,16,21,26,31\}$ so $S_6 = 6 + 11 + 16 + 21 + 26 + 31 = 111$.

Ex. 24: Find the partial sum S_4 for the geometric sequence $\{a_n\} = \left\{10\left(\frac{1}{2}\right)^n\right\}$.

Here, the first 6 terms of the sequence are $\left\{5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}\right\}$ so $S_4 = 5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} = \frac{75}{8}$.

Without going into the proof of it here, there exist formulas for finding the partial sum of both an arithmetic and a geometric sequence.

Definition: The sum S_n of the first n terms of an arithmetic sequence $\{a_n\}$ is given by:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex. 25: Given the arithmetic sequence $\{a_n\} = \{2n - 3\}$

a) Find the partial sum S_{20}.

   We find $a_1 = 2 - 3 = -1$ and $a_{20} = 40 - 3 = 37$. So using the formula, $S_{20} = \frac{20}{2}(-1 + 37) = 360$.

b) Find the formula for the partial sum S_n.

   We have $a_1 = -1$ and $a_n = 2n - 3$. So using the formula, $S_n = \frac{n}{2}(-1 + 2n - 3) = \frac{n}{2}(2n - 4) = \frac{n}{2}(2)(n - 2) = n(n - 2)$.

Definition: The sum S_n of the first n terms of a geometric sequence $\{a_n\}$ with initial (first) term a and ratio $r \neq 1$ is given by:

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Ex. 26: Given the geometric sequence $\{a_n\} = \left\{3,1,\frac{1}{3},\dots\right\}$

a) Find the partial sum S_6.

We have $a = 3$ and $r = \frac{1}{3}$. So using the formula, $S_6 = \frac{3\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \frac{3\left(1-\frac{1}{729}\right)}{\frac{2}{3}} = 3\left(\frac{728}{729}\right)\left(\frac{3}{2}\right) = \frac{364}{81}$.

b) Find the formula for the partial sum S_n.

Given $a = 3$ and $r = \frac{1}{3}$, we have $S_n = \frac{3\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} = \frac{3\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^n}\right)\left(\frac{3}{2}\right) = \frac{9}{2}\left(1-\frac{1}{3^n}\right)$.

Ex. 27: Find the partial sum S_n for the geometric sequence with initial (first) term a and ratio $r = 1$.

Here, $\{a_n\} = \{a, a, a, \dots, a\}$, so we have $S_n = a + a + a + \cdots + a = n \cdot a$.

Notice that this geometric sequence with ratio $r = 1$ is also an arithmetic sequence with $a_1 = a$, $a_n = a$ and common difference $d = 0$. Using the partial sum formula for an arithmetic sequence, we also get $S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(a + a) = \frac{n}{2}(2a) = n \cdot a$

Applications of sequences

Ex. 28 How many numbers divisible by 6 are there between 0 and 200?

The list of numbers divisible by 6 is $\{6,12,18,\dots\}$ which is an arithmetic sequence with $a_1 = 6$ and $d = 6$. The largest number smaller than 200 that is divisible by 6 is 198. We solve for n:

$$a_n = a_1 + (n-1)d$$
$$\to 198 = 6 + (n-1)6 = 6 + 6n - 6$$
$$\to n = 33\ numbers$$

Ex. 29 The seating arrangement in an auditorium has 9 seats in the first row, 13 seats in the second row, and so on, increasing by 4 seats each row for a total of 12 rows.

a) How many seats are in the 8th row?

The sequence of seats is: $\{9,13,17,21,\ldots\}$ which is an arithmetic sequence with $a_1 = 9$ and $d = 4$.

$$a_n = a_1 + (n-1)d = 9 + (n-1)4 = 5 + 4n$$
$$\rightarrow a_8 = 5 + 32 = 38\ seats$$

b) How many seats are there in total in the auditorium?

There are 12 rows in total, so $a_{12} = 9 + (12-1)4 = 9 + 44 = 53$
And the total number of seats is:

$$S_{12} = \frac{12}{2}(a_1 + a_{12}) = 6(9 + 53) = 372\ seats$$

Ex. 30 Assume that the global population of polar bears in 2015 was 26,000 and that it increased yearly by 800. In what year will the global polar bear population reach 36,400?

Here, we have an arithmetic sequence with $a_1 = 26000$ and $d = 800$. Let $a_n = 36400$ and solve for n:

$$a_n = a_1 + (n-1)d \rightarrow 36400 = 26000 + (n-1)(800)$$
$$10400 = (n-1)800$$
$$13 = n - 1$$
$$14 = n$$

So if 2015 represents $n = 1$, then $n = 14$ represents the year 2028.

Ex. 31 Assume that the population of a remote town decreases by 10% every year. If the population was 100,000 at the start of 2015 (year 1), find the projected population at the end of 2030 (year 16).

Here, we have a geometric sequence with $a = a_1 = 100{,}000$ and $r = 1 - 0.1 = 0.9$.
Let $n = 16$ and solve for a_n:

$$a_n = ar^{n-1} = 100{,}000(0.9)^{15} \cong 20{,}589$$

So the projected population in 2030 is approximately 20,589.

<u>Ex. 32</u> A car originally valued at 20,000$ (year 1) depreciates by 15% every year. Find the value of the car at the end of 5 years (or, at the start of the 6th year).

Here, we have a geometric sequence with $a = a_1 = 20{,}000$ and $r = 1 - 0.15 = 0.85$. Let $n = 6$ and solve for a_n:

$$a_n = ar^{n-1} = 20{,}000(0.85)^5 \cong 8{,}874.11$$

So the projected value of the car is 8,874.11$.

<u>Ex. 33</u> A bank loan of 800$ is repaid in annual instalments of 100$ over 8 years, plus 10% interest on the unpaid balance at the start of each year. What is the total amount of interest paid?

Here, we have a finite arithmetic* sequence $\{a_n\}$ where $n = 8$ and the terms of the sequence represent the amount of interest paid at the start of each year.

**It can be shown that sequences representing interest in these types of problems are arithmetic by calculating the interest paid each year..*

At the start, when the loan is borrowed, the unpaid balance is the full loan. So we have the first term $a_1 = 800(0.10) = 80$.

At the start of the last year (the 8th year), there is an unpaid balance of 100$. So the last term $a_8 = 100(0.1) = 10$.

We are being asked to find the total interest paid, meaning the sum of these terms:

$$S_n = \frac{n}{2}(a_1 + a_n) \rightarrow S_8 = \frac{8}{2}(80 + 10) = 360$$

So the total interest paid over 8 years is 360$.

Practice Problems

1. Write the first four terms of the sequence whose general term is given:

   a) $a_n = \frac{3^n}{n^2}$

   b) $a_n = \frac{(-1)^{n+2}}{n^3}$

   c) $a_n = \frac{4^{n-1}}{3^n}$

   d) $a_n = \frac{1}{(n+1)(n+2)}$

2. Determine if the following sequences are arithmetic or geometric. If arithmetic, find d, and if geometric, find r.

   a) $\{7,12,17,\dots\}$

   b) $\{5,10,20,\dots\}$

   c) $\{-1,0,1,\dots\}$

   d) $\{2,-4,8,\dots\}$

   e) $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{8},\dots\right\}$

   f) $\left\{\frac{1}{2},\frac{1}{4},0,\dots\right\}$

   g) $\{27,-18,12,-8,\dots\}$

   h) $\{12,6,0,-6,\dots\}$

   i) $\{1,1.1,1.2,1.3,\dots\}$

3. Find the general term a_n of the following sequences where:

   a) $a_1 = 10$ and $d = -3$

   b) $a_1 = 25$ and $d = 2$

   c) $a_1 = 40$ and $d = -6$

   d) $a_1 = 8$ and $r = \frac{3}{4}$

   e) $a_1 = \frac{1}{2}$ and $r = \frac{2}{3}$

   f) $a_1 = 84$ and $r = \frac{3}{2}$

4. Find the sixth, eleventh, and nth term of the following sequences:

   a) $\{5,9,13,17,\dots\}$

   b) $\{7,2,-3,-8,\dots\}$

   c) $\{4,11,18,25,\dots\}$

   d) $\left\{5,\frac{15}{4},\frac{45}{16},\frac{135}{64},\dots\right\}$

   e) $\left\{2,-1,\frac{1}{2},-\frac{1}{4},\dots\right\}$

   f) $\left\{2,-\frac{4}{3},\frac{8}{9},-\frac{16}{27},\dots\right\}$

5. Determine if the following sequences converge or diverge:

   a) $\{17,24,31,\dots\}$
   b) $\{-5,-1,3,\dots\}$
   c) $\{4,-1,-6,\dots\}$
   d) $\left\{\frac{1}{3},1,3,\dots\right\}$
   e) $\left\{14,2,\frac{2}{7},\dots\right\}$
   f) $\left\{6,3,\frac{3}{2},\dots\right\}$

6. Find the indicated partial sum for each sequence:

   a) Find S_{12} for $\{a_n\}=\{-2,1,4,\dots\}$

   b) Find S_8 for $\{a_n\}=\{3n-4\}$

   c) Find S_9 for $\{a_n\}=\{12,-6,3,\dots\}$

   d) Find S_{15} for $\{a_n\}=\left\{8\left(\frac{1}{2}\right)^n\right\}$

7. Determine how many terms are in the finite arithmetic sequence:

   a) $\{18,24,\dots,336\}$
   b) $\{-4,-1,\dots,224\}$
   c) $\{156,152,\dots,64\}$

8. Determine how many terms are in the finite geometric sequence:

   a) $\left\{\frac{1}{2},2,\dots,512\right\}$
   b) $\{-1,2,\dots,-256\}$
   c) $\{2,6,\dots,486\}$

9. How many numbers are there between 1 and 55 that are:

   a) Divisible by 3?
   b) Divisible by 7?

10. The seating arrangement in a movie theatre has 10 seats in the first row, 15 seats in the second row, and so on, increasing by 5 seats each row for a total of 11 rows.

    a) How many seats are there in the 9th row?
    b) How many seats are there in total?

11. Assume that the North American population of Canada Geese in 2023 was 7,000,000 and that it will increase yearly by 12,000. In what year will the population reach 7,084,000 Canda Geese?

12. Assume that the population of freshwater crab is decreasing by 6% every year. If the population was 323,000,000 in 2023 (year 1), find the projected population in 2030 (year 8).

13. A car originally valued at 80,000$ (year 1) depreciates by 10% every year. Find the value of the car at the end of 7 years (year 8).

14. A bank loan of 6,000$ is repaid in **monthly** instalments of 100$ over 5 **years**, plus 2% interest on the unpaid balance at the start of each **month**. What is the total amount of interest paid?

Answers

1. a) $3, \frac{9}{4}, \frac{27}{9}, \frac{81}{16}$ b) $-1, \frac{1}{8}, -\frac{1}{27}, \frac{1}{64}$ c) $\frac{1}{3}, \frac{4}{9}, \frac{16}{27}, \frac{64}{81}$ d) $\frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \frac{1}{30}$

2. a) $A,\ d = 5$ b) $G\ , r = 2$ c) $A,\ d = 1$ d) $G,\ r = -2$
   e) $G,\ r = \frac{1}{2}$ f) $A,\ d = -\frac{1}{4}$ g) $G, r = -\frac{2}{3}$ h) $A,\ d = -6$
   i) $A,\ d = 0.1$

3. a) $13 - 3n$ b) $2n + 23$ c) $46 - 6n$ d) $8\left(\frac{3}{4}\right)^{n-1}$
   e) $\frac{2^{n-2}}{3^{n-1}}$ f) $7\left(\frac{3^n}{2^{n-2}}\right)$

4. a) $a_6 = 25,\ a_{11} = 45, a_n = 4n + 1$
   b) $a_6 = -18,\ a_{11} = -43,\ a_n = -5n + 12$
   c) $a_6 = 39,\ a_{11} = 74,\ a_n = 7n - 3$
   d) $a_6 = 5\left(\frac{3}{4}\right)^5,\ a_{11} = 5\left(\frac{3}{4}\right)^{10},\ a_n = 5\left(\frac{3}{4}\right)^{n-1}$
   e) $a_6 = -\frac{1}{16}, a_{11} = \frac{1}{512},\ a_n = 2\left(-\frac{1}{2}\right)^{n-1}$
   f) $a_6 = 2\left(-\frac{2}{3}\right)^5,\ a_{11} = 2\left(-\frac{2}{3}\right)^{10}, a_n = 2\left(-\frac{2}{3}\right)^{n-1}$

5. a) D b) D c) D d) D e) C f) C

6. a) 174 b) 76 c) $^{257}/_{32}$ d) 7.999

7. a) 54 b) 77 c) 24

8. a) 6 b) 9 c) 6

9. a) 18 b) 2

10. a) 50 b) 385

11. 2031 12. 209,458,263 13. 38,264$ 14. 3660$