Dawson College

Mathematics Department

Final Examination

201-MA2-DW   Calculus II- Social Science

Winter - 2025

Student Name: _____

Student I.D. #:   _____

Instructor Name: _____

Instructors: G. Bobos-Kristof, G. Honnouvo, E. Richer, N. Sabetghadam,
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INSTRUCTIONS :

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answers use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 15 questions on 15 pages, including this cover page.
- Please <u>ensure</u> that you have a complete exam package before starting.
- The examination must be returned intact.
- A copy of the formula sheet and z-table is provided to you separately.

1. [3+3 marks] A contest offers 35 prizes. The first prize is $125500, and each successive prize is $3250 less than the preceding prize.
   a. What is the value of the twentieth prize?

   b. What is the total amount of money distributed in prizes?

2. [3+3 marks] The bob of a pendulum swings through an arc of 60 inches on its first swing. Each successive swing is 85% of the length of the previous swing. Round off your answer to two decimal places.

   a. Find the length of the 5th swing.

   b. Find the total distance the bob will travel from the beginning until the end of the 5th swing. Round off your answer to two decimal places.

3. [7 marks] Use the limit definition of definite integrals (Riemann Sum) to evaluate the integral. No marks will be given for using antiderivative rules.

$$\int_{1}^{2} (-3 + 4x^2)dx$$

4. [3+3 marks] The daily marginal revenue function is
$$R'(x) = -0.08x + 12$$
   a. Determine the revenue function, if no item sold means no revenue.

   b. Find the revenue of producing and selling 100th to 200th items.

5. [2+3 marks] A certain country's income distribution is described by the function
$$f(x) = \frac{12}{13}x^2 + \frac{1}{13}x$$
   a. Compute $f(0.6)$ and interpret your result.

   b. Find the coefficient of inequality, or Gini Index, of the given Lorenz curve. Round off your answer to four decimal places.

6. [6+6 marks] Evaluate the following indefinite and definite integrals:

   a. $\int \dfrac{10 \ln x}{x^3} dx$

b. $\displaystyle\int_{0}^{5} 3x\sqrt{x+4}\ dx$

7. [6 marks] Evaluate the limit

$$\lim_{x \to 0^+} \left[\frac{2}{x} - \frac{2}{e^x - 1} \right]$$

8. [7 marks] Solve the first-order differential equation by separating the variables.

$$\frac{2 + \tan x}{\sec^2 x} y' = \frac{1}{y}$$

9. [6 marks] Find the area of the region completely enclosed by the graphs of
$f(x) = x^2 + 2x + 1$ and $g(x) = x + 7$.

10. [7 marks] The aerobic rating of a person who is x years old is given by

$$G(x) = -120 \frac{2 - \ln x}{x} \qquad x \geq 10$$

What is a person's average aerobic rating from age 20 to age 30?

11. [6 marks] If the demand function is $p = \dfrac{24}{0.5x+4}$ and the supply function is $p = 4x - 12$, determine the equilibrium and consumers' surplus.

12. [7 marks] Evaluate the improper integral and determine whether it is convergent or divergent.

$$\int_{4}^{\infty} \frac{3e^{-\sqrt{x}}}{\sqrt{x}}\,dx$$

13. [7 marks] In a certain city the population aged 55 years and older (in thousands) from 2010 and 2060 is projected to grow at the rate of

$$P'(t) = \frac{75e^{0.56t}}{e^{0.56t} + 1.768} \qquad 0 \le t \le 5$$

Where t is measured in decades, with $t = 0$ corresponding to 2010. By how much will the population aged 55 years and older increase from the beginning of 2020 until the beginning of 2040.

14. [2+4 marks] Given the probability density function

$$f(x) = \frac{3}{8}x^2, \quad 0 \le x \le 2$$

   a. Find the expected value of the random variable x associated with the probability density function f(x) on [0,2].

   b. Find the variance of the random variable x associated with the probability density function f(x) on [0,2]. Round off your answer to three decimal places.

15. [3+3 marks] A company manufactures electric light bulbs. Laboratory tests show that the lives of these light bulbs are normally distributed with a mean of 700 hr. and a standard deviation of 70 hr. What is the probability that a light bulb selected at random will burn.

a. For more than 840 hr?

b. Between 560 and 770 hr?

ANSWERS:

1. a. 63,750;   b. 2,458,750
2. a. 31.32 in;   b. 222.52 in
3. 19/3
4. a. $R(x) = -0.04x^2 + 12x$;   b. 0
5. a. 0.3785, 60% of the people receive 37.85% of the total income.   b. 4/13~0.3077
6. a. $-\frac{5}{x^2}\ln x - \frac{5}{2x^2} + C$ b. 506/5
7. 1
8. $y = \pm\sqrt{\ln(2 + \tan x)^2 + C}$
9. $\frac{125}{6}u^2$
10. 5.82
11. 3.46
12. $\frac{6}{e^2}$~0.81, C
13. 94,647
14. a. 3/2; b. 3/20
15. a. 0.0228; b. 0.8185