Dawson College

Mathematics Department

Final Examination

Fall 2025

Integral Calculus 201-MA2-DW Social Science

Tuesday, December 16, 2025

14:00 – 17:00

Student Name: ________________________________

Student I.D. #: ________________________________

Instructor Name: ________________________________

Instructors: Georgeana Bobos-Kristof, Ijaz Rajput, Rachid Saad

INSTRUCTIONS:

- Print your name, ID number and instructor's name in the space provided above.
- All questions are to be answered in the space provided for each question.
- Reverse sides of pages may be used for rough work and/or to complete a solution.
- Only calculator permitted is SHARP EL-531***.
- This examination consists of 14 questions, pages 2 through 15. Please ensure that you have a complete examination before starting.

Problem 1. (6 marks)

(a) The selling price of a product increases by a fixed amount each quarter. If the price in the 3rd quarter is \$9.60 and in the 9th quarter is \$27.60, determine the price expected in the 15th quarter.

(b) A promo's quarterly revenue follows a geometric sequence with first term $a_1 = 729$ and common ratio $r = \frac{2}{3}$. Find the sum of the first 6 terms.

(c) The unit production cost is modeled by $a_n = \dfrac{6n^2 - 4n + 9}{3n^2 + 12}$. Determine whether this cost stabilizes as n grows. If so, find the limiting cost.

Problem 2. (6 marks)
The daily marginal profit function is $P'(x) = -0.06x + 15$.
(a) Determine the profit function if producing and selling zero items yields no profit.
(b) Find the profit from producing and selling the 50th to 150th items.

Problem 3. (7 marks)
In a region, the population aged 60 and older (in thousands) grows at the rate

$$P'(t) = \frac{80e^{0.4t}}{e^{0.4t} + 2.5} \qquad 0 \le t \le 50,$$

where t is measured in decades with $t = 0$ corresponding to 2010. By how much will the 60+ population increase from the beginning of 2020 to the beginning of 2040?

Problem 4. (7 marks)
Use the limit definition of definite integrals (Riemann Sum) to evaluate the integral. No marks will be given for using antiderivative rules.

$$\int_0^3 \left(2x + 5x^2\right)\, dx$$

Problem 5. (9 marks)
Evaluate the following:

$$\text{(a)} \int_1^4 \frac{x}{\sqrt{x^2+3}}\,dx \qquad \text{(b)} \int \frac{12\,\ln x}{x^4}\,dx$$

Problem 6. (6 marks)
Find the area of the region completely enclosed by the graphs of $f(x) = x^2 - 4x + 5$ and $g(x) = x + 1$.

Problem 7. (6 marks)

A country's income distribution is modeled by the Lorenz curve $L(x) = \dfrac{10}{11}x^2 + \dfrac{1}{11}x$.

(a) Compute $L(0.7)$ and interpret the result.

(b) Find the Gini index of this Lorenz curve, rounded to four decimal places.

Problem 8. (7 marks)

A company's efficiency after x hours is modeled by $T(x) = 300\,\dfrac{6 - \ln x}{x}$, for $x \geq 1$. Find the average efficiency on the interval $[8, 500]$.

Problem 9. (7 marks)
If the demand function is $p = 36.40 - 0.85x$ and the supply function is $p = 12.80 + 1.15x$, determine the equilibrium and the consumers' surplus.

Problem 10. (6 marks)
Evaluate the limit:

$$\lim_{x\to 0}\left(\frac{3}{\sin x}-\frac{3}{x}\right)$$

Problem 11. (7 marks)
Solve the differential equation by separating variables:

$$\frac{\cos(2y)}{e^{x^2}}y' = x$$

given $y(0) = 0$

Problem 12. (7 marks)
Evaluate the improper integral and determine whether it converges or diverges:

$$\int_{16}^{\infty} \frac{9e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

Problem 13. (9 marks)
Given the probability density function

$$f(x) = \frac{3}{4}x(2-x), \qquad 0 \leq x \leq 2,$$

(a) find the expected value $E[X]$.
(b) find the variance $\mathrm{Var}(X)$, rounding to three decimal places.

Problem 14. (6 marks)
A company produces laptop batteries whose lifetimes are normally distributed with a mean of 4800 hours and a standard deviation of 450 hours.
(a) What is the probability that a randomly selected battery lasts longer than 5500 hours?
(b) What is the probability that a battery lasts between 4200 and 5100 hours?

Answers

Problem 1

(a) Arithmetic sequence: $a_3 = 9.60$, $a_9 = 27.60$.

$$d = \frac{27.60 - 9.60}{9 - 3} = 3, \qquad a_1 = 9.60 - 2(3) = 3.60, \qquad a_{15} = 3.60 + 14(3) = 45.60$$

(b) Geometric sequence: $a_1 = 729$, $r = \frac{2}{3}$.

$$S_6 = 729 \cdot \frac{1 - \left(\frac{2}{3}\right)^6}{1 - \frac{2}{3}} = 729 \cdot \frac{1 - \frac{64}{729}}{\frac{1}{3}} = 665 \times 3 = 1995$$

(c) $\displaystyle\lim_{n\to\infty} \frac{6n^2 - 4n + 9}{3n^2 + 12} = \frac{6}{3} = \$2/\text{unit}$. Yes, the cost stabilizes.

Problem 2

(a) $P(x) = \int(-0.06x + 15)\,dx = -0.03x^2 + 15x + C$. $\quad P(0) = 0 \Rightarrow C = 0$.

$$P(x) = -0.03x^2 + 15x$$

(b) $\displaystyle\int_{50}^{150} P'(x)\,dx = P(150) - P(50) = 1575 - 675 = \$900$

Problem 3

$2020 \to t = 1$, $\quad 2040 \to t = 3$. $\quad$ Let $u = e^{0.4t} + 2.5$, $\quad du = 0.4\,e^{0.4t}\,dt$.

$$\int_1^3 \frac{80\,e^{0.4t}}{e^{0.4t} + 2.5}\,dt = 200\ln\left(e^{0.4t} + 2.5\right)\Big|_1^3 \approx 75{,}414 \text{ (thousands)}$$

Problem 4 – Riemann Sum

$\Delta x = \dfrac{3}{n}$, $\quad x_k = \dfrac{3k}{n}$.

$$\int_0^3 (2x + 5x^2)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left[2 \cdot \frac{3k}{n} + 5\left(\frac{3k}{n}\right)^2\right] \frac{3}{n} = \lim_{n\to\infty} \left[\frac{18}{n^2}\sum k + \frac{135}{n^3}\sum k^2\right]$$

Using $\sum k = \frac{n(n+1)}{2}$ and $\sum k^2 = \frac{n(n+1)(2n+1)}{6}$:

$$= \lim_{n\to\infty} \left[\frac{18\,n(n+1)}{2n^2} + \frac{135\,n(n+1)(2n+1)}{6n^3}\right] = 9 + 45 = 54$$

Problem 5

(a) Let $u = x^2 + 3$, $\quad du = 2x\,dx$. When $x = 1$, $u = 4$; when $x = 4$, $u = 19$.

$$\int_1^4 \frac{x}{\sqrt{x^2 + 3}}\,dx = \frac{1}{2}\int_4^{19} u^{-1/2}\,du = \left[\sqrt{u}\right]_4^{19} = \sqrt{19} - 2 \approx 2.36$$

(b) Integration by parts: $u = 12\ln x$, $\quad dv = x^{-4}\,dx$; $\quad du = \frac{12}{x}\,dx$, $\quad v = -\frac{1}{3x^3}$.

$$\int \frac{12\ln x}{x^4}\,dx = -\frac{4\ln x}{x^3} - \frac{4}{3x^3} + C$$

Problem 6

Intersections: $x^2 - 4x + 5 = x + 1 \;\Rightarrow\; x^2 - 5x + 4 = 0 \;\Rightarrow\; x = 1,\ 4.$

$$A = \int_1^4 \big[(x+1) - (x^2 - 4x + 5)\big]\,dx = \int_1^4 (-x^2 + 5x - 4)\,dx = 4.5$$

Problem 7

(a) $L(0.7) = \frac{10}{11}(0.49) + \frac{1}{11}(0.7) = \frac{5.6}{11} \approx 0.5091$

Interpretation: the bottom 70% of the population earns approximately 50.91% of total income.

(b) $G = 2\int_0^1 \big[x - \frac{10}{11}x^2 - \frac{1}{11}x\big]\,dx = \dfrac{20}{11}\displaystyle\int_0^1 (x - x^2)\,dx = \dfrac{20}{11}\cdot\dfrac{1}{6} = \dfrac{10}{33} \approx 0.3030$

Problem 8

$$T_{\text{avg}} = \frac{1}{500-8}\int_8^{500} \frac{300(6-\ln x)}{x}\,dx. \quad \text{Let } u = 6 - \ln x, \quad du = -\tfrac{dx}{x}.$$

$$= -\frac{150}{492}\,(6-\ln x)^2\Big|_8^{500} \approx 4.672$$

Problem 9

Equilibrium: $36.40 - 0.85x = 12.80 + 1.15x \;\Rightarrow\; \bar{x} = 11.8, \quad \bar{p} = 26.37.$

$$\text{CS} = \int_0^{11.8} D(x)\,dx - \bar{p}\cdot\bar{x} \approx \$59.18$$

Problem 10

$$\lim_{x\to 0}\left(\frac{3}{\sin x} - \frac{3}{x}\right) = 3\lim_{x\to 0}\frac{x - \sin x}{x\sin x} \overset{\text{L'H}}{=} 3\lim_{x\to 0}\frac{1-\cos x}{\sin x + x\cos x} \overset{\text{L'H}}{=} 3\lim_{x\to 0}\frac{\sin x}{2\cos x - x\sin x} = \frac{0}{2} = 0$$

Problem 11

Separate: $\cos(2y)\,dy = x\,e^{x^2}\,dx$. Integrate both sides:

$$\frac{\sin(2y)}{2} = \frac{e^{x^2}}{2} + C$$

$y(0) = 0$: $\quad 0 = \frac{1}{2} + C \;\Rightarrow\; C = -\frac{1}{2}.\quad$ So $\sin(2y) = e^{x^2} - 1.$

$$y = \tfrac{1}{2}\arcsin\!\Big(e^{x^2} - 1\Big)$$

Problem 12

Let $u = -\sqrt{x}, \quad \frac{dx}{\sqrt{x}} = -2\,du.$

$$\int_{16}^{\infty} \frac{9\,e^{-\sqrt{x}}}{\sqrt{x}}\,dx = -18\,e^{-\sqrt{x}}\Big|_{16}^{\infty} = 0 + 18\,e^{-4} \approx 0.3297$$

The integral converges.

Problem 13

(a) $E[X] = \frac{3}{4}\int_0^2 (2x^2 - x^3)\,dx = \frac{3}{4} \cdot \frac{4}{3} = 1$

(b) $E[X^2] = \frac{3}{4}\int_0^2 (2x^3 - x^4)\,dx = \frac{3}{4} \cdot \frac{8}{5} = \frac{6}{5} = 1.2$

$\mathrm{Var}(X) = E[X^2] - (E[X])^2 = 1.2 - 1 = 0.200$

Problem 14

(a) $z = \frac{5500 - 4800}{450} = 1.56.$ $\quad P(X > 5500) = 1 - 0.9406 = 0.0594\ (5.94\%)$

(b) $z_1 = \frac{4200 - 4800}{450} = -1.33, \quad z_2 = \frac{5100 - 4800}{450} = 0.67.$

$P(4200 < X < 5100) = 0.7486 - 0.0918 = 0.6568\ (65.68\%)$