Dawson College
Mathematics Department
FINAL EXAMINATION
Probability and Statistics for Social Science
201-MA4-DW section 00001 and 00002

Thursday, May 21, 2026

Student Name:_____

Student I.D#:_____

Instructor: Mehdi Moodi

Time: 9:30 - 12:30

Instructions:

- Print your name and ID in the provided space.
- This exam contains 10 pages (including this cover page) and 11 questions for a total of 100 points. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Show all steps and justify your answers where required.
- If an exact answer is not possible, round your answer to the nearest 10000th.
- You are only permitted to use the Sharp EL-531** calculator.
- This examination booklet must be returned intact.
- Good luck!

Problem 1. /8 points

At a college cafeteria, 500 students were observed to see whether they bought a coffee and whether they bought a pastry on a certain morning.

	Pastry	No Pastry	Total
Coffee	120	180	300
No Coffee	80	120	200
Total	200	300	500

Let C be the event "the student bought a coffee" and let P be the event "the student bought a pastry."

(a) Find $P(C \cup P)$. /2

Brief solution: $P(C \cup P) = P(C) + P(P) - P(C \cap P) = \frac{300}{500} + \frac{200}{500} - \frac{120}{500} = 0.7600.$

(b) Find $P(C \mid P)$. /2

Brief solution: $P(C \mid P) = \frac{P(C \cap P)}{P(P)} = \frac{120/500}{200/500} = \frac{120}{200} = 0.6000.$

(c) Find $P(P \mid C)$. /2

Brief solution: $P(P \mid C) = \frac{P(C \cap P)}{P(C)} = \frac{120/500}{300/500} = \frac{120}{300} = 0.4000.$

(d) Determine whether C and P are independent. Justify your answer algebraically. /2

Brief solution: $P(C \cap P) = \frac{120}{500} = 0.2400$ and $P(C)P(P) = \frac{300}{500} \cdot \frac{200}{500} = 0.6000(0.4000) = 0.2400.$
Since these are equal, C and P are independent.

Problem 2. /6 points

A campus bakery buys croissants from three suppliers.

- 50% come from Supplier A, and 2% of those are stale.
- 30% come from Supplier B, and 5% of those are stale.
- 20% come from Supplier C, and 8% of those are stale.

A croissant is chosen at random.

(a) Find the probability that the croissant is stale. /2

Brief solution: $P(S) = 0.50(0.02) + 0.30(0.05) + 0.20(0.08) = 0.0100 + 0.0150 + 0.0160 = 0.0410.$

(b) Given that the croissant is stale, find the probability that it came from Supplier B. /4

Brief solution: $P(B \mid S) = \dfrac{P(B \cap S)}{P(S)} = \dfrac{0.30(0.05)}{0.0410} = \dfrac{0.0150}{0.0410} = 0.3659.$

Problem 3. /8 points

Let X be the number of late buses observed during one hour at a certain stop. Its probability distribution is given below:

x	0	1	2	3	4
$P(X = x)$	0.10	0.25	k	0.30	0.15

(a) Find k. /2

Brief solution: Probabilities add to 1, so $k = 1 - (0.10 + 0.25 + 0.30 + 0.15) = 0.20$.

(b) Find $E(X)$. /2

Brief solution: $E(X) = 0(0.10) + 1(0.25) + 2(0.20) + 3(0.30) + 4(0.15) = 2.1500$.

(c) Find $\text{Var}(X)$. /3

Brief solution: $E(X^2) = 0^2(0.10) + 1^2(0.25) + 2^2(0.20) + 3^2(0.30) + 4^2(0.15) = 6.1500$. Then $\text{Var}(X) = E(X^2) - [E(X)]^2 = 6.1500 - (2.1500)^2 = 1.5275$.

(d) Find σ_X. /1

Brief solution: $\sigma_X = \sqrt{1.5275} = 1.2359$.

Problem 4. /10 points

The weight X (in grams) of boxes of breakfast cereal is normally distributed with mean $\mu = 510$ and standard deviation $\sigma = 12$.

(a) Find $P(X < 528)$. /3

Brief solution: $z = \dfrac{528 - 510}{12} = 1.50$. So $P(X < 528) = P(Z < 1.50) = 0.9332$.

(b) Find the value c such that 90% of the boxes weigh less than c. /3

Brief solution: Need the 90th percentile, so $z \approx 1.2816$. Then $c = 510 + 1.2816(12) = 525.3792$.

(c) A random sample of $n = 36$ boxes is selected. Let $\overline{X}$ be the sample mean weight. Find $P(\overline{X} < 506)$. /4

Brief solution: $\mu_{\overline{X}} = 510$, $\sigma_{\overline{X}} = \dfrac{12}{\sqrt{36}} = 2$. Then $z = \dfrac{506 - 510}{2} = -2.00$. So $P(\overline{X} < 506) = P(Z < -2.00) = 0.0228$.

Problem 5. /8 points

A survey is conducted to estimate the true proportion p of Dawson students who support extending library hours during exam period. In a random sample of 250 students, 142 say that they support the change.
Construct a 99% confidence interval for p. Be sure to verify that the normal approximation is appropriate.

Brief solution: $\hat{p} = \dfrac{142}{250} = 0.5680$, so $\hat{q} = 0.4320$. Conditions: $n\hat{p} = 142 \geq 5$ and $n\hat{q} = 108 \geq 5$, so the normal approximation is appropriate. For a 99% confidence interval, use $z_{0.005} = 2.575$. Standard error:

$$\sqrt{\frac{(0.5680)(0.4320)}{250}} = 0.0313.$$

Margin of error:

$$E = 2.575(0.0313) = 0.0807.$$

Therefore, the 99% confidence interval is

$$0.5680 \pm 0.0807 = (0.4873,\ 0.6487).$$

Problem 6. /8 points

A random sample of 16 repair times (in minutes) for a certain machine is selected. The sample mean is $\bar{x} = 42.3$ minutes and the sample standard deviation is $s = 5.6$ minutes. Assume the population of repair times is normally distributed.
Construct a 95% confidence interval for the true mean repair time μ.

Brief solution: Since $n = 16$ and σ is unknown, use a t interval with $df = 15$. For 95%, $t_{0.025,15} \approx 2.131$. Standard error: $\dfrac{5.6}{\sqrt{16}} = 1.4000$. Margin of error: $2.131(1.4000) = 2.9834$. CI: $42.3 \pm 2.9834 = (39.3166,\ 45.2834)$.

Problem 7. /10 points

A bottling company claims that the mean amount of juice in its bottles is 750 mL. A random sample of 36 bottles is selected, giving $\bar{x} = 746.8$ mL and $s = 7.2$ mL.
Use these data to test, at the 5% level of significance, whether the true mean amount of juice is less than 750 mL. Measure the strength of evidence against the null hypothesis by calculating the p-value.

Brief solution: $H_0 : \mu = 750$, $H_a : \mu < 750$. Since $n = 36 > 30$, use a z test. Test statistic: $z = \dfrac{746.8 - 750}{7.2/\sqrt{36}} = \dfrac{-3.2}{1.2} = -2.6667$. The p-value is $P(Z < -2.6667) \approx 0.0038$. Since $0.0038 < 0.05$, reject H_0. There is sufficient evidence that the true mean amount is less than 750 mL.

Problem 8. /10 points

A university administrator claims that at most 40% of students skip breakfast on class days. In a random sample of 200 students, 94 say that they skip breakfast on class days.
Use these data to test, at the 2% level of significance, whether the true proportion of students who skip breakfast is different from 40%. Be sure to verify that the large-sample normal test is appropriate.

Brief solution: $H_0 : p = 0.40$, $H_a : p \neq 0.40$. $\hat{p} = \dfrac{94}{200} = 0.4700$. Check conditions under H_0: $np_0 = 200(0.40) = 80 \geq 5$ and $nq_0 = 200(0.60) = 120 \geq 5$, so the large-sample normal test is appropriate. Test statistic:

$$z = \frac{0.4700 - 0.4000}{\sqrt{(0.40)(0.60)/200}} = 2.0207.$$

For a two-tailed test, the p-value is

$$2P(Z > 2.0207) \approx 2(0.0217) = 0.0434.$$

Since $0.0434 > 0.02$, fail to reject H_0. There is not sufficient evidence at the 2% level to conclude that the true proportion of students who skip breakfast is different from 40%.

Problem 9. /12 points

A city planner claims that the average travel time for commuters who use the express bus is **more than 5 minutes less** than the average travel time for commuters who use the regular bus.
Let

μ_1 = mean travel time for express-bus commuters, μ_2 = mean travel time for regular-bus commuters.

Independent random samples were taken, with the following results:

- Express bus: $n_1 = 49,\ \bar{x}_1 = 31.4,\ s_1 = 6.3$
- Regular bus: $n_2 = 64,\ \bar{x}_2 = 39.0,\ s_2 = 7.2$

Use these data to test, at the 5% level of significance, whether the city planner's claim is supported. Measure the strength of evidence against the null hypothesis by calculating the p-value.

Brief solution: The claim is $\mu_1 - \mu_2 < -5$. So $H_0 : \mu_1 - \mu_2 = -5$, $H_a : \mu_1 - \mu_2 < -5$. Since both samples are large, use a z test. Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (-5)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(31.4 - 39.0) - (-5)}{\sqrt{6.3^2/49 + 7.2^2/64}} = \frac{-2.6}{1.2728} = -2.0428.$$

The p-value is $P(Z < -2.0428) \approx 0.0205$. Since $0.0205 < 0.05$, reject H_0. There is sufficient evidence to support the city planner's claim.

Problem 10. /8 points

A quality-control inspector knows that 8% of light bulbs produced by a certain machine are defective. A random sample of 12 light bulbs is selected.
Let X be the number of defective bulbs in the sample.

(a) Explain why X can be modeled by a binomial random variable, and state the values of n and p. /1

Brief solution: Yes. There is a fixed number of trials, two outcomes for each bulb, independent trials, and constant probability of defect. Thus $X \sim \text{Bin}(12, 0.08)$, so $n = 12$ and $p = 0.08$.

(b) Find the probability that exactly 2 of the 12 bulbs are defective. /3

Brief solution: $P(X = 2) = \binom{12}{2}(0.08)^2(0.92)^{10} = 0.1835$.

(c) Find the probability that at most 1 of the 12 bulbs is defective. /2

Brief solution: $P(X \leq 1) = P(X = 0) + P(X = 1) = (0.92)^{12} + \binom{12}{1}(0.08)(0.92)^{11} = 0.7513$.

(d) Find the mean and standard deviation of X. /2

Brief solution: $\mu = np = 12(0.08) = 0.9600$ and $\sigma = \sqrt{npq} = \sqrt{12(0.08)(0.92)} = 0.9398$.

Problem 11. /12 points

A company wants to know whether a training program reduces the time required for workers to complete a certain task. The same 8 workers were timed before and after the training program.

Worker	Before (min)	After (min)
1	42	39
2	38	36
3	45	40
4	50	44
5	41	40
6	47	43
7	39	37
8	44	41

Let the paired difference be defined as

$$d = \text{Before} - \text{After}.$$

Assume that the population of paired differences is approximately normal.

(a) Find the 8 paired differences. /2

Brief solution: The paired differences are 3, 2, 5, 6, 1, 4, 2, 3.

(b) Using your calculator, find the sample mean $\bar{d}$ and the sample standard deviation s_d of the paired differences. /4

Brief solution: Using the 8 differences, $\bar{d} = 3.2500$ and $s_d = 1.6690$.

(c) Test, at the 5% level of significance, whether the training program decreases the mean completion time. Clearly state the null and alternative hypotheses. /6

Brief solution: Since $d = \text{Before} - \text{After}$, a decrease in completion time means $\mu_d > 0$. So $H_0 : \mu_d = 0$ and $H_a : \mu_d > 0$. Test statistic:

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{3.2500}{1.6690/\sqrt{8}} = 5.5076.$$

With $df = 7$, the critical value for a right-tailed test at $\alpha = 0.05$ is about 1.895. Since $5.5076 > 1.895$, reject H_0. There is sufficient evidence that the training program decreases the mean completion time.