DAWSON COLLEGE
Mathematics Department

FINAL EXAMINATION

Calculus I- 201-NYA-05 (Open/Commerce)
Fall 2021

Instructor: Noushin Sabetghadam

Student Name: ______________________________

Student ID. #: ______________________________

Instructions:

- Print your name and ID in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly and indicate your final answer(s).
- Only calculators Sharp EL 531.X/ XG/XT are permitted.
- This examination booklet must be returned intact.

This examination consists of 11 questions. Please ensure that you have a complete examination booklet before starting.

(1) Evaluate the following limits if possible and **write all the details.**(No marks is given if you use L'hopital Rules)

(a) (**5 marks**) $\lim_{x\to 1} \frac{x - \frac{1}{x}}{x^2 - 8x + 7}$

(b) (**4 marks**) $\lim_{x\to 0} \frac{\sin(2x)\ \tan(3x)}{4x^2}$

(c) (**6 marks**) $\lim_{x\to\ -3} \frac{\sqrt{x^2+7} - 4}{x^2 + 3x}$

(2) (**5 marks**) Use only the limit definition of the derivative to evaluate $f'(x)$ where

$$f(x) = 3x^2 - 2x - 5$$

(3) (**4 marks**) **(a)** Evaluate For which value(s) of a the function f is continuous everywhere?

$$f(x) = \begin{cases} ax^2 + x & if\ x \leq 1 \\ \dfrac{3a}{x+1} & if\ x > 1 \end{cases}$$

(**3 marks**) **(b)** If $\lim_{x \to 2} f(x) = 1$ and $\lim_{x \to 2} g(x) = -3$, evaluate:

$$\lim_{x \to 2} \frac{f(x) - 2g(x)}{x + \sqrt{f(x)}}$$

(4) (**20 marks**) Find the derivative. (**Do NOT simplify**)

(a) $y = \left(\dfrac{2x^3 + 5x}{\sqrt{3x^5 + 2x}} \right)^3$

(b) $f(x) = \cos^2(x^2 + 3) + 3\tan(\log_2 x)$

(c) $y = (1 + \ln x)^{\sec x}$

(d) $g(x) = 3^{(2x+1)} \cdot \arctan(x^2)$

(5) If $sin(y) + 3x^2y^3 = 5x - y$, then:

(a) (**5 marks**) find $\dfrac{dy}{dx}$

(b) (**2 marks**) Find the tangent line to the graph of the given function at the point $(0, 0)$.

(6) For the given total cost and the demand function,

$$C(x) = 16000 + 500x - 1.6x^2 + 0.004x^3, \qquad p = 1700 - 0.7x$$

(a) (**3 marks**) Find the profit function.

(b) (**2 marks**) Find the actual profit/loss of producing and selling 105-th item.

(c) (**2 marks**) Find the marginal profit which gives an estimation for the profit of producing and selling 105-th unit.

(7) (**6 marks**) Suppose a police officer is 1/2 mile south of an intersection, driving north towards the intersection at 32 mph. At the same time, another car is 1/2 mile east of the intersection, driving east (away from the intersection) at an unknown speed. The officer's radar gun indicates 12 mph when pointed at the other car (that is, the straight-line distance between the officer and the other car is increasing at a rate of 12 mph). What is the speed of the other car?

(8) **(6 marks)** A rectangular closed storage container is to have a base whose length is two times its width and a volume of 1440cm^3. If the material for the base and top costs \$0.25 per cm^2 and the material for the sides costs \$0.10 per cm^2, find the cost for the cheapest such container.

(9) **(5 marks)** Find the point(s) on the graph of the function $y = \dfrac{x^3}{(x^2-1)^2}$ where the tangent line is horizontal.

(10) **(5 marks)** Let $f(x) = \dfrac{\ln x}{x}$, Find $f''(x)$ and then evaluate $f''(e)$.

(11) Given the function $f(x) = \dfrac{x^2 - 2}{4 - x^2}$.

We also know that $f'(x) = \dfrac{4x}{(4 - x^2)^2}$ and $f''(x) = \dfrac{12x^2 + 16}{(4 - x^2)^3}$.

(a) **(2 marks)** Find the domain of f.

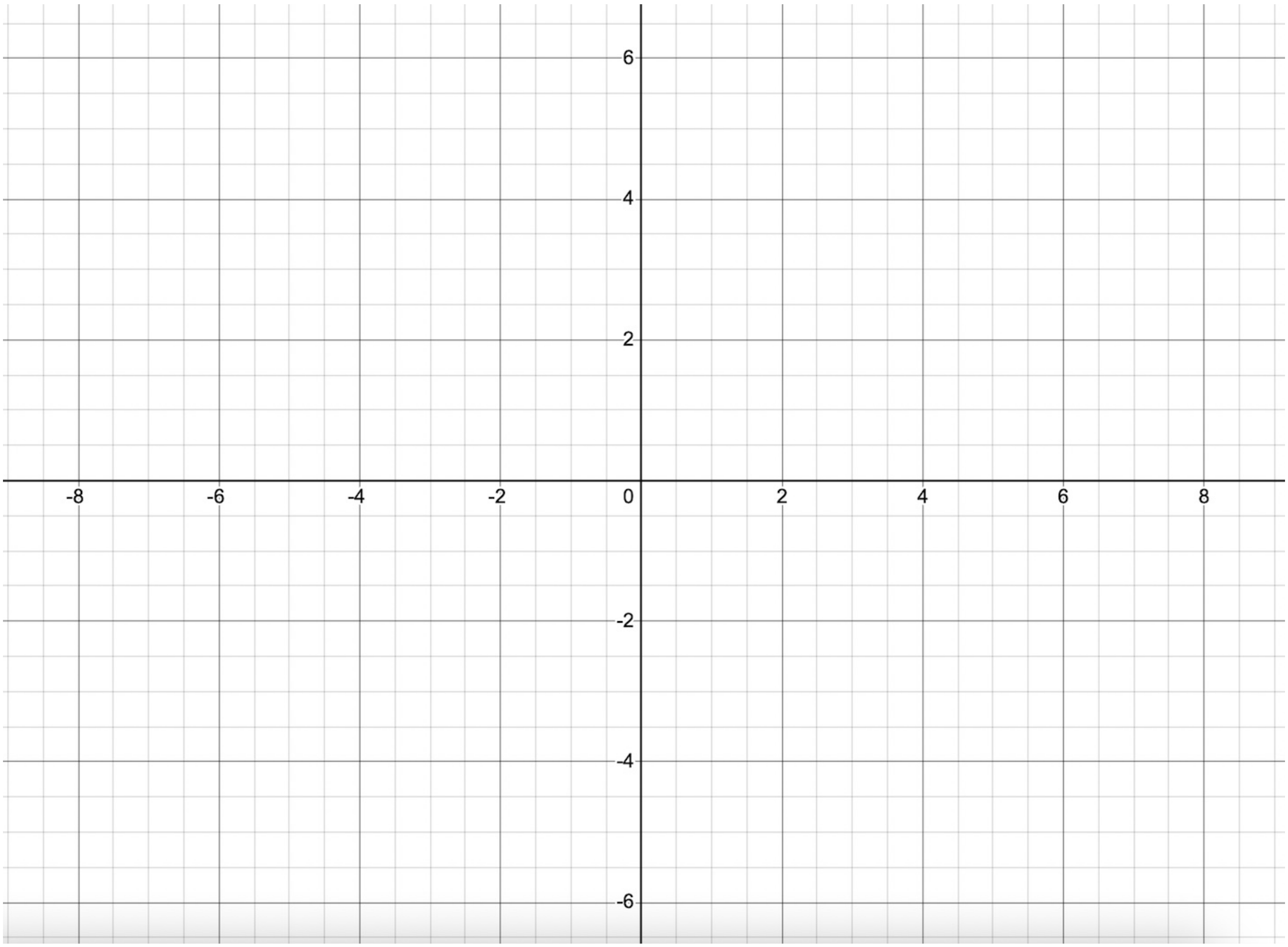
(b) **(3 marks)** Find the $x-$intercept(s) and the $y-$intercept of f.

(c) **(4 marks)** Find the vertical and the horizontal asymptotes of the function f and justify your answers.

(d) **(3 marks)** Find the **intervals** where the function f is increasing or decreasing and also find its **relative maximum point(s) and relative minimum point(s)** if there is any.

(e) **(3 marks)** Find the **intervals** where the function f is concave upward or downward and also find its **inflection point(s)** if there is any.

(f) **(2 marks)** Use all the data collected about f to draw the its graph in the coordinate system below.



Final Answers

(1) (a) $\dfrac{-1}{3}$

(b) $\dfrac{3}{2}$

(c) $\dfrac{1}{4}$

(2) $f'(x) = 6x - 2$

(3) (a) $a = 2$

(b) $\dfrac{7}{3}$

(4) **(a)** $y' = 3\left(\dfrac{2x^3+5x}{\sqrt{3x^5+2x}}\right)^2 \dfrac{(6x^2+5)(\sqrt{3x^5+2x} - \frac{15x^4+2}{2\sqrt{3x^5+2x}}(2x^3+5x)}{(3x^5+2x)}$

(b) $f'(x) = -2\cos(x^2+3)2x\sin(x^2+3) + 3\sec^2(\log_2 x)\dfrac{1}{x\ln 2}$

(c) $y' = (1+\ln x)^{\sec x}[\sec x \tan x \ln(1+\ln x) + \sec x \dfrac{\frac{1}{x}}{1+\ln x}]$

(d) $g'(x) = 2\ln 3 \cdot 3^{(2x+1)} \cdot \arctan(x^2) + \dfrac{2x}{1+x^2} \cdot 3^{(2x+1)}$

(5) (a) $y' = \dfrac{5 - 6xy^3}{\cos y + 9x^2y^2 + 1}$

(b) $y = \dfrac{5}{2}x$

(6) (a) $P(x) = -0.004x^3 + 0.9x^2 + 1200x - 16000$

(b) 1257.06\$

(c) 1257.41\$

(7) $\dfrac{dx}{dt} \approx 48.97$ mph

(8) 108\$

(9) at (0.0)

(10) $f''(x) = \dfrac{2\ln x - 3}{x^3}$, $f''(e) = \dfrac{-1}{e^3}$

(11) (a) $D(f) = \mathbb{R} \setminus \{-2, 2\}$

(b) $(-\sqrt{2}, 0)$, $(\sqrt{2}, 0)$, $(0, -1/2)$

(c) $x = -2$, $x = 2$, $y = -1$

(d) f is increasing on $(0,2) \cup (2, \infty)$ and decreasing on $(-\infty, -2) \cup (-2, 0)$. There is a relative min at $(0, -1/2)$.

(e) f is concave upward on $(-2, 2)$ and downward on $(-\infty, -2) \cup (2, \infty)$. There is no inflection point.

(f)