201-NYA-05 Science

FALL 2014

1. Evaluate the following limits *without* using L'Hôpital's rule. Give exact answers (no decimals).

   (a) (4 marks) $\lim_{x\to 2} \dfrac{3x^2-5x-2}{12-3x^2}$ $= -\dfrac{7}{12}$

   (b) (4 marks) $\lim_{x\to -1} \dfrac{\frac{1}{x+4}-\frac{1}{3}}{x^3+1}$ $= -\dfrac{1}{27}$

   (c) (4 marks) $\lim_{x\to 4^+} \dfrac{2e^{x-4}}{16-x^2}$ $= -\infty$

2. For the following piece-wise defined function $f(x) = \begin{cases} \frac{x^2-3x+2}{|x-2|} & x < 2 \\ 0 & x = 2 \\ \frac{6x}{x^2-16} & x > 2 \end{cases}$

(a) (4 marks) Find $\lim_{x \to 2} f(x)$ if it exists.

-1

(b) (2 marks) Find any x-value(s) where $f(x)$ is discontinuous.

$x = 4$

$x = 2$

3. (5 marks) Use only the limit definition of the derivative to find $f'(x)$ for $f(x) = \sqrt{5-x}$. No marks will be given for using the differentiation rules.

$$f'(x) = \frac{-1}{2\sqrt{5-x}}$$

4. Find y': (Do not simplify your answer.)

(a) (4 marks) $y = \dfrac{e^{\csc x}}{\sqrt{x^2+1}}$

$$y' = \frac{\sqrt{x^2+1}\, e^{\csc x}(-\csc x \cot x) - e^{\csc x}\frac{1}{2}(x^2+1)^{-\frac{1}{2}} 2x}{\left(\sqrt{x^2+1}\right)^2}$$

(b) (4 marks) $y = (x^4+1)^6 \arctan(3x) + \pi^2$

$$y' = (x^4+1)^6 \frac{1}{1+(3x)^2} 3 + \arctan(3x)\, 6(x^4+1)^5(4x^3)$$

(c) (4 marks) $y = 3^{\arcsin(\log_2(x^2+1))}$

$$y' = (\ln 3)\, 3^{\arcsin(\log_2(x^2+1))} \cdot \frac{1}{\sqrt{1-(\log_2(x^2+1))^2}} \cdot \frac{1}{(\ln 2)(x^2+1)}(2x)$$

Question 4 continued.

(d) (4 marks) $y = (\sin x)^{\cos x}$

$$y' = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - (\sin x)\ln(\sin x)\right]$$

(e) (4 marks) $y = \sqrt[3]{\sec(\tan^5(2x))}$

$$y' = \frac{1}{3}\left(\sec(\tan^5(2x))\right)^{-\frac{2}{3}} \sec(\tan^5(2x))\tan(\tan^5(2x))5\tan^4(2x)\cdot$$
$$\sec^2(2x)(2)$$

5. (4 marks) Find the equation of the tangent line to the graph of the relation $3e^{xy} - x = 0$ at the point $(3, 0)$.

$y = \frac{1}{9}x - \frac{1}{3}$

6. An object suspended from a spring is oscillating so that its displacement from equilibrium as a function of time t in seconds is given by $y(t) = 0.8\cos(10t)$ centimeters.
(Round your answers to 2 decimal places.)

(a) (2 marks) Find the velocity v(t) of the object, and calculate v(2).

$-8\sin(10t)$

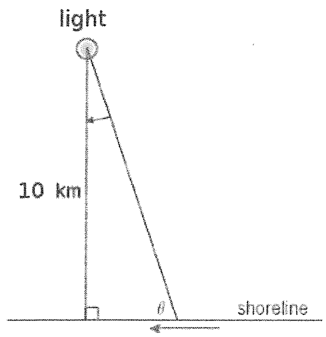
-7.30 cm/sec

(b) (2 marks) Find the acceleration a(t) of the object, and calculate a(2).

$-80\cos(10t)$

-32.65 cm/sec^2

7. (5 marks) A revolving light, located 10 km from a straight shoreline, turns at a constant angular speed of 3 rad/min. How fast is the spot of the light moving along the shore when the beam makes an angle of $\frac{\pi}{3}$ radians with the shoreline?



-40 km/min

8. Evaluate the following limits.

(a) (4 marks) $\lim_{x\to 0^+}\left(\frac{1}{3x}-\frac{1}{3x\cos 3x}\right)$ $= 0$

(b) (4 marks) $\lim_{x\to 0^+} x^{\frac{\ln 5}{1+\ln x}}$ $= 5$

9. (5 marks) Find the absolute maximum and absolute minimum of the function $f(x) = (x^2 - 1)^{\frac{2}{3}}$ on the interval [-3,0].

(-1,0) abs min.

(-3,4) abs max

10. Let $f(x) = \dfrac{x^2}{(x-1)^2}$, $f'(x) = \dfrac{-2x}{(x-1)^3}$, $f''(x) = \dfrac{4x+2}{(x-1)^4}$.

For the function $f(x)$:

(a) (1 mark) Find the x and y intercepts.

(0,0)

(b) (2 marks) Find any horizontal and vertical asymptotes.

H.A : y = 1

V.A : x = 1

Question 10 continued.

$$f(x) = \frac{x^2}{(x-1)^2},\ f'(x) = \frac{-2x}{(x-1)^3},\ f''(x) = \frac{4x+2}{(x-1)^4}.$$

(c) (3 marks) Find any local/relative maxes/mins and the intervals of increase and decrease.

$(0,0)$ local min.

(d) (3 marks) Find any points of inflection and the intervals of concavity.

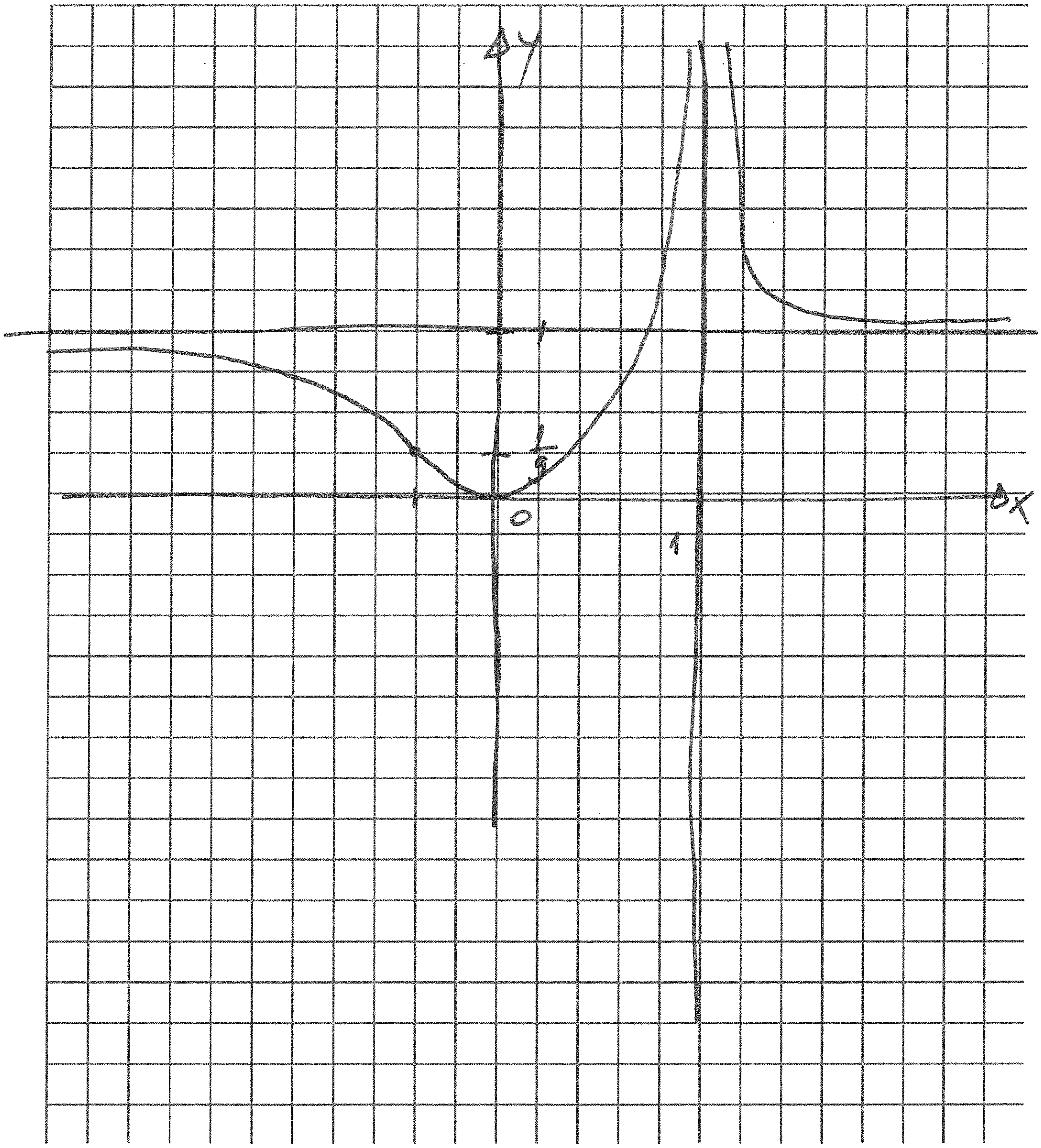
$\left(-\frac{1}{2}, \frac{1}{9}\right)$

concave up $\left(-\frac{1}{2}, 1\right)$

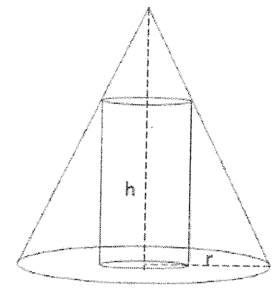
— " — down $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$

Question 10 continued.

(e) (2 marks) Sketch the graph of $f(x)$, labelling the important points you have found.



11. (5 marks) Find the radius and height of the right circular cylinder of maximum volume that can be inscribed in a right circular cone with radius 6 centimeters and height 10 centimeters. ($V_{cylinder} = \pi r^2 h$)



$r = 4$ cm

$h = \frac{10}{3}$ cm

12. Evaluate the following indefinite integrals.

(a) (4 marks) $\int \frac{1+\sqrt[3]{x}+x\sin x}{x}dx$

$= \ln|x| + 3\sqrt[3]{x} - \cos x + C$

(b) (4 marks) $\int \frac{\csc^2 x}{\sqrt{1+\cot x}}dx$

$= -2\sqrt{1+\cot x} + C$

13. (4 marks) Find the particular solution of the differential equation $(x^2+4)^3\frac{dy}{dx} = \frac{x}{2y^2}$ given the condition $y(1) = 2$. (You may leave your answer in implicit form.)

$$y^3 = \frac{-3}{8(x^2+4)^2} + \frac{1603}{200}$$

14. (3 marks) Find all the values of n such that $y = x^n$ satisfies the differential equation $x^2y'' - 2xy' = 4y$ for all real values of x.

$n = 4$

$n = -1$