201-NYB-05
Calculus 2 (Science)
Final Exam

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Name: SOLUTIONS

Student ID: ____________________

- There are a total of **100 marks** on this test. There are 13 pages **excluding** the cover pages.
- Show all your work where indicated. Incomplete or unjustified answers will not receive full marks. Clearly indicate your final answers.
- You may use the back of the pages to show your work if you run out of room. Please clearly indicate where your work may be found.
- Do not remove any pages from the exam booklet.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Marks	5	4	25	5	5	7	10	5	4	15	5	5	5	100

1. (a) Find $\int_{-1}^{1}(4-x^2)\,dx$ by taking the limit of a Riemann Sum. **[4 marks]**

(b) Check your answer using the Fundamental Theorem of Calculus (a.k.a. the Evaluation Theorem). **[1 mark]**

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

a) $\Delta x = \frac{2}{n}$, $x_i^* = -1 + \frac{2i}{n}$

$$\int_{-1}^{1}(4-x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(4-\left(-1+\frac{2i}{n}\right)^2\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(4-\left(1-\frac{4i}{n}+\frac{4i^2}{n^2}\right)\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(3+\frac{4i}{n}-\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}(3n)+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}-\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty}\left(6+\frac{8}{2}\cdot\frac{n^2+n}{n^2}-\frac{8}{6}\cdot\frac{2n^3+3n^2+n}{n^3}\right)$$

$$= 6+4(1)-\frac{4}{3}(2) = 10-\frac{8}{3} = \boxed{\frac{22}{3}}$$

b) $$\int_{-1}^{1}(4-x^2)\,dx = \left[4x-\frac{x^3}{3}\right]_{-1}^{1} = \left(4-\frac{1}{3}\right)-\left(-4+\frac{1}{3}\right)$$

$$= 4-\frac{1}{3}+4-\frac{1}{3}$$

$$= \boxed{\frac{22}{3}}$$

2. If $\int_3^5 (f(x)-1)\,dx = 5$ and $\int_3^0 2f(x)\,dx = 3$, find $\int_0^5 f(x)\,dx$. **[4 marks]**

$$\int_3^5 (f(x)-1)\,dx = 5 \Rightarrow \int_3^5 f(x)\,dx - (5-3) = 5$$

$$\Rightarrow \int_3^5 f(x)\,dx = 7$$

$$\int_3^0 2f(x)\,dx = 3 \Rightarrow \int_0^3 f(x)\,dx = -\frac{3}{2}$$

$$\therefore \int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx = -\frac{3}{2}+7 = \boxed{\frac{11}{2}}$$

3. Evaluate each integral (continued on next two pages).

(a) $\int \sin^5\theta \cot^2\theta \, d\theta$ **[5 marks]**

$$= \int \sin^5\theta \cdot \frac{\cos^2\theta}{\sin^2\theta} \, d\theta$$

$$= \int \sin^3\theta \cos^2\theta \, d\theta$$

$$= \int \sin^2\theta \cos^2\theta \sin\theta \, d\theta$$

$$= \int (1-\cos^2\theta)\cos^2\theta \sin\theta \, d\theta$$

$u = \cos\theta \, d\theta$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$$= -\int (1-u^2)u^2 \, du$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C = \boxed{\frac{1}{5}\cos^5\theta - \frac{1}{3}\cos^3\theta + C}$$

(b) $\int_0^1 x \arctan x \, dx$ **[5 marks]**

By parts: $u = \arctan x$ $\quad dv = x\,dx$

$du = \frac{1}{1+x^2}dx$ $\quad v = \frac{1}{2}x^2$

$$\left[\frac{1}{2}x^2 \arctan x\right]_0^1 - \int_0^1 \frac{1}{2}\frac{x^2}{1+x^2}\,dx$$

$$= \left[\frac{1}{2}(1)^2\frac{\pi}{4} - 0\right] - \frac{1}{2}\int_0^1 \frac{x^2+1-1}{1+x^2}\,dx$$

$$= \frac{\pi}{8} - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[x - \arctan x\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[\left(1-\frac{\pi}{4}\right) - (0-0)\right] = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \boxed{\frac{\pi}{4} - \frac{1}{2}}$$

(continued on next page)

3. (continued)

(c) $\displaystyle\int \frac{2x+1}{x^3+6x^2+9x}\,dx$ [5 marks]

$$= \int \frac{2x+1}{x(x+3)^2}\,dx$$

$$= \int \left(\frac{1/9}{x} - \frac{1/9}{x+3} + \frac{5/3}{(x+3)^2} \right) dx$$

$$= \frac{1}{9}\ln|x| - \frac{1}{9}\ln|x+3| - \frac{5}{3}(x+3)^{-1} + C$$

or

$$\boxed{\frac{1}{9}\ln\left|\frac{x}{x+3}\right| - \frac{5}{3(x+3)} + C}$$

$$\frac{2x+1}{x(x+3)^2} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$2x+1 = A(x+3)^2 + Bx(x+3) + Cx$$

$$2x+1 = A(x^2+6x+9) + Bx^2 + 3Bx + Cx$$

$$2x+1 = Ax^2 + 6Ax + 9A + Bx^2 + 3Bx + Cx$$

$$A + B = 0$$

$$6A + 3B + C = 2$$

$$9A = 1$$

$$\Rightarrow A = \frac{1}{9}$$

$$B = -\frac{1}{9}$$

$$C = 2 - 6\left(\frac{1}{9}\right) - 3\left(-\frac{1}{9}\right)$$

$$= 2 - \frac{2}{3} + \frac{1}{3} = \frac{5}{3}$$

(continued on next page)

3. (continued)

(d) $\int \frac{x^2}{(9-x^2)^{3/2}}dx$ $\qquad$ $x = 3\sin\theta$, $dx = 3\cos\theta \, d\theta$ $\qquad$ [5 marks]

$$= \int \frac{9\sin^2\theta}{(9-9\sin^2\theta)^{3/2}} 3\cos\theta \, d\theta$$

$$= \int \frac{27\sin^2\theta\cos\theta}{(9\cos^2\theta)^{3/2}} d\theta$$

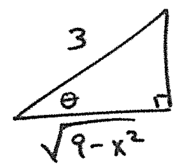
$$= \int \frac{27\sin^2\theta\cos\theta}{27\cos^3\theta} d\theta$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta} d\theta$$

$$= \int \tan^2\theta \, d\theta$$

$$= \int (\sec^2\theta - 1) d\theta \longrightarrow = \tan\theta - \theta + C$$

$\sin\theta = \frac{x}{3}$



$$= \boxed{\frac{x}{\sqrt{9-x^2}} - \arcsin\left(\frac{x}{3}\right) + C}$$

(e) $\int \frac{1+\sqrt{t}}{1-\sqrt{t}}dt$ $\qquad$ [5 marks]

$u = 1 - \sqrt{t}$

$u - 1 = -\sqrt{t}$ or $\sqrt{t} = 1 - u$

$t = (u-1)^2$

$dt = 2(u-1)du$

$$= \int \frac{1+(1-u)}{u} \cdot 2(u-1)du$$

$$= 2\int \frac{(2-u)(u-1)}{u} du$$

$$= 2\int \frac{-u^2+3u-2}{u} du$$

$$= 2\int \left(-u + 3 - \frac{2}{u}\right) du$$

$$= 2\left(-\frac{1}{2}u^2 + 3u - 2\ln|u|\right) + C$$

$$= \boxed{-(1-\sqrt{t})^2 + 6(1-\sqrt{t}) - 4\ln|1-\sqrt{t}| + C}$$

4. Find the average value of the function $g(x) = e^{-x}\cos(1 - e^{-x})$ over the interval $[-\ln(1+\pi), 0]$. [5 marks]

$$g_{avg} = \frac{1}{0-(-\ln(1+\pi))}\int_{-\ln(1+\pi)}^{0} e^{-x}\cos(1-e^{-x})\,dx$$

$$= \frac{1}{\ln(1+\pi)}\int_{-\pi}^{0}\cos(u)\,du$$

$$= \frac{1}{\ln(1+\pi)}\Big[\sin(u)\Big]_{-\pi}^{0}$$

$$= \frac{1}{\ln(1+\pi)}(0-0)$$

$$= \boxed{0}$$

$u = 1 - e^{-x}$

$du = e^{-x}\,dx$

$x = 0 \Rightarrow u = 1 - e^{0} = ($

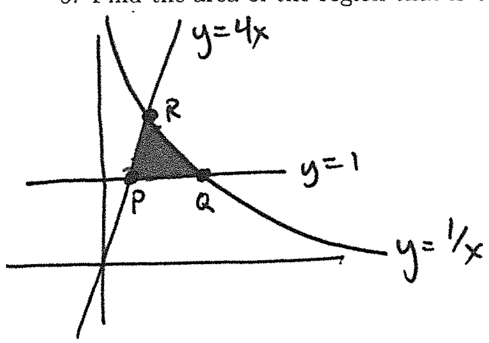
$x = -\ln(1+\pi)$

$\Rightarrow u = 1 - e^{\ln(1+}$

$= 1-(1+\pi)$

$= -\pi$

5. Find the area of the region that is bounded by the graphs of $y = 4x$, $y = \frac{1}{x}$, and $y = 1$. [5 marks]



$P:\ 4x = 1 \Rightarrow x = \frac{1}{4}$

$Q:\ \frac{1}{x} = 1 \Rightarrow x = 1$

$R:\ \frac{1}{x} = 4x \Rightarrow 1 = 4x^2 \Rightarrow x = 1/2$

$$A = \int_{1/4}^{1/2}(4x-1)\,dx + \int_{1/2}^{1}\left(\frac{1}{x}-1\right)dx$$

$$= \Big[2x^2 - x\Big]_{1/4}^{1/2} + \Big[\ln|x| - x\Big]_{1/2}^{1}$$

$$= \left(2\cdot\frac{1}{4}-\frac{1}{2}\right) - \left(2\cdot\frac{1}{16}-\frac{1}{4}\right) + (0-1) - \left(\ln\frac{1}{2}-\frac{1}{2}\right)$$

$$= \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - \frac{1}{8} + \frac{1}{4} - 1 - \ln\frac{1}{2} + \frac{1}{2}$$

$$= \ln 2 - \frac{3}{8}.$$

<u>OR</u>: $A = \int_{1}^{2}\left(\frac{1}{y} - \frac{1}{4}y\right)dy = \Big[\ln|y| - \frac{1}{8}y^2\Big]_{1}^{2} = \left(\ln 2 - \frac{1}{2}\right) - \left(0 - \frac{1}{8}\right)$

$= \ln 2 - \frac{3}{8}.$

6. Let $\mathcal{R}$ be the region bounded by $y = \sin x$ and $y = -\sin x$ from $x = 0$ to $x = \frac{\pi}{2}$.

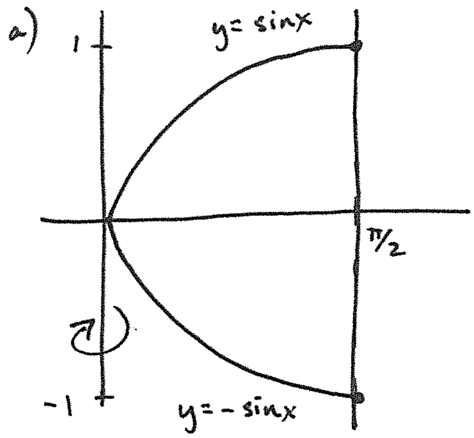
(a) **[3+3 marks]** Set up the integral for the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis:

i. by using the "shell" method **[3 marks]**

ii. by using the "disc/washer" method **[3 marks]**

(b) Determine the volume by evaluating ONE of the integrals from part (a). **[1 mark]**

a)



i) Shell:

$$V = \int_0^{\pi/2} 2\pi x \left(\sin x - (-\sin x)\right) dx$$

$$= \int_0^{\pi/2} 4\pi x \sin x \, dx$$

ii) Washers

$y = \sin x \iff x = \arcsin y$

$y = -\sin x \iff x = \arcsin(-y)$

$$V = \int_{-1}^{0} \pi\left[\left(\frac{\pi}{2}\right)^2 - (\arcsin(-y))^2\right] dy + \int_0^1 \pi\left[\left(\frac{\pi}{2}\right)^2 - (\arcsin y)^2\right] dy$$

or, by symmetry,

$$V = 2\int_0^1 \pi\left(\left(\frac{\pi}{2}\right)^2 - (\arcsin y)^2\right) dy$$

b) $$V = 4\pi \int_0^{\pi/2} x \sin x \, dx$$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$$= 4\pi\left[-x\cos x\right]_0^{\pi/2} + 4\pi \int_0^{\pi/2} \cos x \, dx$$

$$= 4\pi\left[-\frac{\pi}{2}(0) + 0\right] + 4\pi\left[\sin x\right]_0^{\pi/2}$$

$$= 0 + 4\pi[1 - 0] = 4\pi$$

7. Determine the convergence of each improper integral. Determine the value of the integral if possible.

(a) $\displaystyle\int_0^{\pi/2} \left(\frac{1}{x - \frac{\pi}{2}} - \tan x \right) dx$ [5 marks]

$$= \lim_{t \to \frac{\pi}{2}^-} \int_0^t \left(\frac{1}{x - \pi/2} - \tan x \right) dx$$

$$= \lim_{t \to \frac{\pi}{2}^-} \Big[\ln|x - \tfrac{\pi}{2}| - \ln|\sec x| \Big]_0^t$$

$$= \lim_{t \to \frac{\pi}{2}^-} \Big[\ln|t - \tfrac{\pi}{2}| - \ln|\sec t| \Big] - \ln \tfrac{\pi}{2} + \ln 1$$

Since $|t - \frac{\pi}{2}| \to 0^+ \Rightarrow \ln|t - \frac{\pi}{2}| \to -\infty$

$|\sec t| \to \infty \Rightarrow \ln|\sec t| \to \infty$

$\therefore \displaystyle\int_0^{\pi/2} \left(\frac{1}{x - \pi/2} - \tan x \right) dx = -\infty$ $\therefore$ diverges.

(b) $\displaystyle\int_1^{\infty} \frac{2x+5}{x^2+1}\, dx$ [5 marks]

$$= \lim_{t \to \infty} \int_1^t \Big(\underbrace{\frac{2x}{x^2+1}}_{\substack{u = x^2+1 \\ du = 2x\,dx}} + \frac{5}{x^2+1} \Big) dx$$

$$= \lim_{t \to \infty} \Big[\ln|x^2+1| + 5 \arctan x \Big]_1^t$$

$$= \lim_{t \to \infty} \Big(\underbrace{\ln(t^2+1)}_{\to \infty} + \underbrace{5 \arctan t}_{\to \pi/2} - \ln 2 - 5(\pi/4) \Big)$$

$$= \infty$$

$\therefore$ diverges.

8. Find a general formula for the n^{th} term of the following sequence (which begins with $n = 1$), assuming the pattern continues:

$$\left\{ \frac{\sin\left(\frac{\pi}{7}\right)}{2}, \frac{\sin\left(\frac{\pi}{8}\right)}{6}, \frac{\sin\left(\frac{\pi}{9}\right)}{10}, \frac{\sin\left(\frac{\pi}{10}\right)}{14}, \dots \right\}.$$

Does this sequence converge or diverge? Justify your answer. **[5 marks]**

$$a_n = \frac{\sin\left(\frac{\pi}{n+6}\right)}{4n-2}$$

$$-1 \leq \sin\left(\frac{\pi}{n+6}\right) \leq 1 \implies \frac{-1}{4n-2} \leq a_n \leq \frac{1}{4n-2}$$

Since $\lim_{n\to\infty} \frac{-1}{4n-2}$ and $\lim_{n\to\infty} \frac{1}{4n-2}$ both $= 0$,

$\lim_{n\to\infty} a_n = 0$ by the Squeeze Theorem

9. Determine if the series $\sum_{n=1}^{\infty} \frac{5e^{n+1}}{2^{3n}}$ converges or diverges. If it converges, find its sum. **[4 marks]**

$$\sum_{n=1}^{\infty} \frac{5e^2 e^{n-1}}{8^n} = \sum_{n=1}^{\infty} \frac{5e^2}{8} \cdot \left(\frac{e}{8}\right)^{n-1}$$

Geometric series with $a = \frac{5e^2}{8}$, $r = \frac{e}{8} < 1$

$\therefore$ converges.

$$S = \frac{a}{1-r} = \frac{5e^2/8}{1-e/8} \cdot \frac{8}{8} = \boxed{\frac{5e^2}{8-e}}$$

10. Determine if each series is absolutely convergent, conditionally convergent, or divergent. Clearly state which test you are using for each problem.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\ln(n^2+1)}$ **[5 marks]**

Test for divergence:

$$\lim_{n\to\infty} \frac{\ln n}{\ln(n^2+1)} = \lim_{n\to\infty} \frac{1/n}{2n/n^2+1}$$

$$= \lim_{n\to\infty} \frac{1}{n}\cdot\frac{n^2+1}{2n}$$

$$= \lim_{n\to\infty} \frac{n^2+1}{2n^2}$$

$$= \frac{1}{2} \neq 0 \quad \therefore \text{ divergent.}$$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3-\arctan n}{\sqrt{n^4+1}}$ (all positive terms) **[5 marks]**

$\boxed{\text{I}}$ Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2}$ (convergent p-series)

$$3-\arctan(n) \leq 3$$

$$\Rightarrow \frac{3-\arctan(n)}{\sqrt{n^4+1}} \leq \frac{3}{\sqrt{n^4+1}} \leq \frac{3}{\sqrt{n^4}} = \frac{3}{n^2} \qquad \therefore \sum_{n=1}^{\infty} \frac{3-\arctan n}{\sqrt{n^4+1}}$$

is absolutely convergent.

$\boxed{\text{II}}$ Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{\frac{3-\arctan n}{\sqrt{n^4+1}}}{1/n^2} = \lim_{n\to\infty} (3-\arctan n)\cdot\frac{n^2}{\sqrt{n^4+1}}$$

$$= \lim_{n\to\infty} (3-\arctan n)\sqrt{\frac{n^4}{n^4+1}} = \left(3-\frac{\pi}{2}\right)(1) > 0$$ (continued on next page)

$$\therefore \sum_{n=1}^{\infty} \frac{3-\arctan(n)}{\sqrt{n^4+1}} \text{ is abs. convergent}$$

10. (continued)

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2n)!}{5^n \cdot (n!)^2}$ **[5 marks]**

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}(2n+2)!}{5^{n+1}((n+1)!)^2} \cdot \frac{5^n (n!)^2}{(-1)^n (2n)!} \right|$$

$$= \lim_{n\to\infty} \frac{5^n}{5^{n+1}} \cdot \frac{(2n+2)!}{(2n)!} \cdot \frac{n!}{(n+1)!} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} \frac{1}{5} \cdot \frac{(2n+2)(2n+1)}{1} \cdot \frac{1}{n+1} \cdot \frac{1}{n+1}$$

$$= \lim_{n\to\infty} \frac{4n^2+6n+2}{5n^2+10n+5}$$

$$= \lim_{n\to\infty} \frac{4+6/n+2/n^2}{5+10/n+5/n^2} = \frac{4}{5} < 1$$

$\therefore$ absolutely convergent.

11. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n^2}$. **[5 marks]**

Ratio test:

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-3)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n(x-3)^n} \right|$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^2 |x-3|$$

$$= (1)|x-3| = |x-3|$$

Convergent if $|x-3|<1$, or $2<x<4$.

If $x=2$, then

$$\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$ which is a convergent p-series.

If $x=4$, then

$$\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n^2} = \sum_{n=1}^{\infty} \frac{-1}{n^2}.$$

Since $\sum_{n=1}^{\infty} \left|\frac{-1}{n^2}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$, this series is absolutely convergent.

$\therefore$ interval of convergence is $[2,4]$.

12. (a) Find the Taylor series for $f(x) = \ln x$ centered at $x = 1$.
Note: Assume that $f(x)$ has a power series expansion; do not show that $R_n(x) \to 0$. **[4 marks]**

$$f(x) = \ln x$$
$$f'(x) = x^{-1}$$
$$f''(x) = -x^{-2}$$
$$f'''(x) = 2x^{-3}$$
$$f^{(4)}(x) = -2\cdot 3x^{-4}$$
$$f^{(5)}(x) = 2\cdot 3\cdot 4x^{-5}$$
$$f^{(n)}(x) = (-1)^{n-1}(n-1)!\,x^{-n} \quad \text{if } n \geq 1$$

$$\Rightarrow f^{(n)}(1) = \begin{cases} 0 & \text{if } n = 0 \\ (-1)^{n-1}(n-1)! & \text{if } n \geq 1 \end{cases}$$

$\therefore$ the Taylor series is

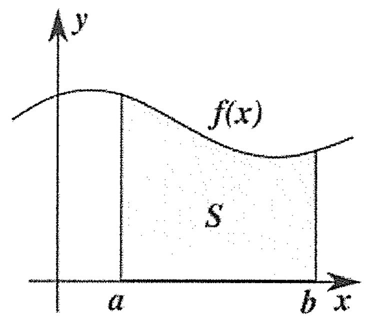
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n = 0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n-1)!}{n!}(x-1)^n$$
$$= \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x-1)^n}$$

(b) Use part (a) to show that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n2^n} = \ln\frac{3}{2}$.
Note: You may assume that $f(x) = \ln x$ converges to its Taylor series if $0 < x < 2$. **[1 mark]**

By letting $x = \frac{3}{2}$, we have

$$\ln\frac{3}{2} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(\frac{3}{2}-1\right)^n$$
$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n2^n}.$$

13. Let $f(x)$ be a continuous function on the interval $[a, b]$, where $0 < a < b$, and let S be the region bounded by $f(x)$, $x = a$, $x = b$ and the x-axis.



Suppose the volume of the solid formed by revolving S about the y-axis is 4π and the volume of the solid formed by revolving S about the line $x = -1$ is 8π. Find the area of S. **[5 marks]**

$$4\pi = \int_a^b 2\pi x f(x)\,dx \implies \int_a^b x f(x)\,dx = 2$$

$$8\pi = \int_a^b 2\pi (x+1) f(x)\,dx \implies \int_a^b (x f(x) + f(x))\,dx = 4$$

So

$$\int_a^b (x f(x) + f(x))\,dx - \int_a^b x f(x)\,dx = 4 - 2$$

$$\implies \int_a^b (\cancel{x f(x)} + f(x) - \cancel{x f(x)})\,dx = 2$$

$$\implies \underbrace{\int_a^b f(x)\,dx}_{= \text{area of } S} = 2$$

$\therefore$ the area is 2.