201-NYB-05
Calculus 2 (Science)
Final Exam

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Instructors:
Mahendra Chaubey, Oxana Diaconescu, Laine Frajberg, Igor Gorelyshev,
Alexander Hariton, Yann Lamontagne, Noushin Sabetghadam,
Benjamin Seamone, Shahab Shahabi, Bogdan Szczepara

Name: SOLUTIONS (A)

Student ID: ____________________

- There are a total of **100 marks** on this test.
- There are a total of **12 pages** on this test, not including the cover pages.
- Show all your work unless indicated otherwise. Incomplete or unjustified answers will not receive full marks.
- You may use the back of the pages to show your work if you run out of room.
- Do not remove any pages from the exam booklet.

1. Find $\int_{-4}^{2}\left(-3-\frac{3}{2}x\right)\,dx$ by

[4 pt] (a) taking the limit of a Riemann Sum;

[2 pt] (b) interpreting the integral in terms of areas.

Summation formulas:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

(a) $\Delta x = \frac{6}{n}$, $x_i = -4+\frac{6i}{n}$

$$\int_{-4}^{2}\left(-3-\frac{3}{2}x\right)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(-3-\frac{3}{2}\left(-4+\frac{6i}{n}\right)\right)\frac{6}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(-3+6-\frac{9i}{n}\right)\frac{6}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{18}{n}-\frac{54i}{n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{18}{n}(n)-\frac{54}{n^2}\cdot\frac{n(n+1)}{2}\right)$$

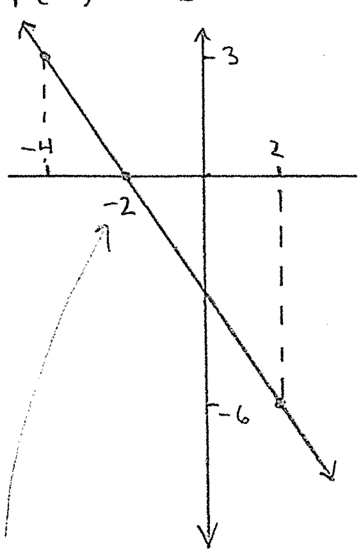
$$= 18-27$$

$$= -9$$

(b) $f(x) = -3-\frac{3}{2}x$

$f(-4) = 3$

$f(2) = -6$



$-3-\frac{3}{2}x = 0$

$\Rightarrow x = -2$

$$\int_{-4}^{2} f(x)\,dx = \frac{1}{2}(2)(3)-\frac{1}{2}(4)(6)$$

$$= 3-12$$

$$= -9$$

2. Evaluate each integral

[5 pt] (a) $\displaystyle\int \frac{1}{(9+t^2)^{3/2}}\,dt$

Trig substitution: $t = 3\tan\theta$, $dt = 3\sec^2\theta\, d\theta$

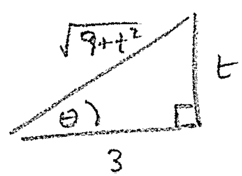
$$\int \frac{1}{(9+t^2)^{3/2}}\,dt = \int \frac{3\sec^2\theta\, d\theta}{(9+9\tan^2\theta)^{3/2}} = \int \frac{3\sec^2\theta\, d\theta}{(9\sec^2\theta)^{3/2}} = \int \frac{3\sec^2\theta}{27\sec^3\theta}\,d\theta$$

$$= \frac{1}{9}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{9}\int \cos\theta\, d\theta = \frac{1}{9}\sin\theta + C$$

$$= \frac{1}{9}\cdot\frac{t}{\sqrt{9+t^2}} + C$$

$t = 3\tan\theta$

$\Rightarrow \tan\theta = \frac{t}{3}$



[5 pt] (b) $\displaystyle\int 3x^2\sin^2(x^3)\cos^2(x^3)\,dx$

Let $u = x^3$, $du = 3x^2\,dx$:

$$\int 3x^2\sin^2(x^3)\cos^2(x^3)\,dx = \int \sin^2(u)\cos^2(u)\,du = \int \left(\frac{1-\cos 2u}{2}\right)\left(\frac{1+\cos 2u}{2}\right)du$$

$$= \frac{1}{4}\int (1-\cos^2 2u)\,du = \frac{1}{4}\int \sin^2 2u\,du = \frac{1}{4}\int \left(\frac{1-\cos 4u}{2}\right)du$$

$$= \frac{1}{8}\left(u - \frac{1}{4}\sin 4u\right) + C = \frac{1}{8}x^3 - \frac{1}{32}\sin 4x^3 + C$$

[5 pt] (c) $\int \frac{1}{v(v+3)^2}\,dv = \int \left(\frac{A}{v} + \frac{B}{v+3} + \frac{C}{(v+3)^2} \right) dv$

$1 = A(v+3)^2 + Bv(v+3) + Cv$

$v=0 \implies 1 = 9A \implies A = 1/9$

$v=-3 \implies 1 = -3C \implies C = -1/3$

$v=-2 \implies 1 = A - 2B - 2C \implies 1 = \frac{1}{9} - 2B + \frac{2}{3}$

$\implies 2B = \frac{1}{9} + \frac{6}{9} - \frac{9}{9} = \frac{-2}{9} \implies B = -\frac{1}{9}.$

$$\int \frac{1}{v(v+3)^2}\,dv = \int \left(\frac{1/9}{v} - \frac{1/9}{v+3} - \frac{1/3}{(v+3)^2} \right) dv$$

$$= \frac{1}{9}\ln|v| - \frac{1}{9}\ln|v+3| + \frac{1}{3}\cdot\frac{1}{v+3} + C$$

$$= \ln \sqrt[9]{\left|\frac{v}{v+3}\right|} + \frac{1}{3(v+3)} + C$$

[5 pt] (d) $\int e^{\sqrt{3x+1}}\,dx$

$t = \sqrt{3x+1}$

$t^2 = 3x+1$

$2t\,dt = 3dx$

$dx = \frac{2}{3}t\,dt$

$= \int e^t \cdot \frac{2}{3} t\,dt$

$= \frac{2}{3}\int te^t\,dt$

By parts $\longrightarrow$

$u = t \qquad du = dt$

$dv = e^t dt \qquad v = e^t$

$= \frac{2}{3}\left(te^t - \int e^t\,dt\right)$

$= \frac{2}{3}te^t - \frac{2}{3}e^t + C$

$= \frac{2}{3}\sqrt{3x+1}\,e^{\sqrt{3x+1}} - \frac{2}{3}e^{\sqrt{3x+1}} + C$

[5 pt] 3. Find the average value of the function $f(\theta) = \tan^3(3\theta)$ on the interval $\left[0, \frac{\pi}{12}\right]$.

$$f_{avg} = \frac{1}{\frac{\pi}{12} - 0} \int_0^{\pi/12} \tan^3(3\theta)\,d\theta$$

$$= \frac{12}{\pi} \int_0^{\pi/12} \tan(3\theta)\tan^2(3\theta)\,d\theta$$

$$= \frac{12}{\pi} \int_0^{\pi/12} \tan(3\theta)\left(\sec^2(3\theta) - 1\right) d\theta$$

$$= \frac{12}{\pi} \int_0^{\pi/12} \underbrace{\tan(3\theta)}_{u}\, \underbrace{\sec^2(3\theta)\,d\theta}_{\frac{1}{3}du} - \frac{12}{\pi} \int_0^{\pi/12} \tan(3\theta)\,d\theta$$

$$= \frac{12}{\pi}\left[\frac{1}{2}\tan^2(3\theta)\cdot\frac{1}{3}\right]_0^{\pi/12} - \frac{12}{\pi}\left[\frac{1}{3}\ln|\sec(3\theta)|\right]_0^{\pi/12}$$

$$= \frac{2}{\pi}\left[\tan^2(\pi/4) - \tan^2(0)\right] - \frac{4}{\pi}\left[\ln|\sec \pi/4| - \ln|\sec 0|\right]$$

$$= \frac{2}{\pi}\left[(1)^2 - (0)^2\right] - \frac{4}{\pi}\left[\ln\sqrt{2} - \ln 1\right]$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\ln\sqrt{2} = \frac{2}{\pi}(1 - \ln 2).$$

[5 pt] 4. Find the value of the integral $\int_0^1 x^3 \ln x\,dx$ or show it diverges.

Discontinuous at $x = 0$:

By parts

$u = \ln x \qquad du = \frac{1}{x}dx$

$dv = x^3 dx \qquad v = \frac{1}{4}x^4$

$$\lim_{t\to 0^+} \int_t^1 x^3 \ln x\,dx$$

$$= \lim_{t\to 0^+}\left[\frac{1}{4}x^4\ln x - \int \frac{1}{4}x^4\cdot\frac{1}{x}dx\right]_t^1$$

$$= \lim_{t\to 0^+}\left[\frac{1}{4}x^4\ln x - \int \frac{x^3}{4}dx\right]_t^1$$

$$= \lim_{t\to 0^+}\left[\frac{1}{4}x^4\ln x - \frac{x^4}{16}\right]_t^1 = \lim_{t\to 0^+}\left[\left(0 - \frac{1}{16}\right) - \left(\underbrace{\frac{1}{4}t^4\ln t}_{\to(0)(-\infty)} - \underbrace{\frac{t^4}{16}}_{\to 0}\right)\right]$$

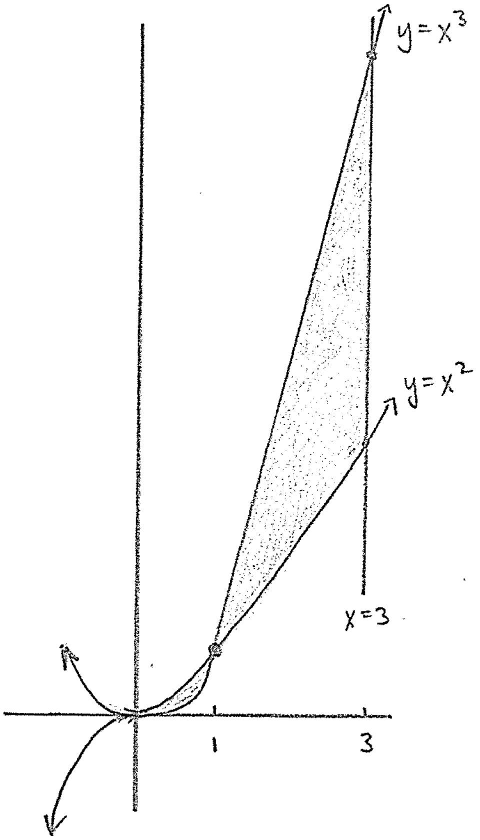
$$= -\frac{1}{16} - \lim_{t\to 0^+}\frac{\ln t}{4t^{-4}}$$

$$= -\frac{1}{16} - \lim_{t\to 0^+}\frac{1/t}{-16t^{-5}}$$

$$= -\frac{1}{16} + \lim_{t\to 0^+}\frac{t^4}{16} = -\frac{1}{16} + 0 = -\frac{1}{16}$$

$\therefore$ the integral converges to $-\frac{1}{16}$.

[5 pt] 5. Determine the total area of the region(s) bounded by the following: $y = x^3$, $y = x^2$, $x = 3$.



$$A = \int_0^1 (x^2 - x^3)\,dx + \int_1^3 (x^3 - x^2)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^3$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right) - (0 - 0) + \left(\frac{81}{4} - 9\right) - \left(\frac{1}{4} - \frac{1}{3}\right)$$

$$= \frac{4 - 3 + 243 - 108 - 3 + 4}{12}$$

$$= \frac{137}{12}$$

[5 pt] 6. Find the length of the curve $y = \frac{1}{2}(e^x + e^{-x})$ on the interval $0 \le x \le \ln 2$.

$$y' = \frac{1}{2}(e^x - e^{-x}) \implies (y')^2 = \frac{1}{4}(e^{2x} - 2 + e^{-2x}) = \frac{e^{2x}}{4} - \frac{1}{2} + \frac{e^{-2x}}{4}$$

$$L = \int_0^{\ln 2} \sqrt{1 + (y')^2}\,dy = \int_0^{\ln 2} \sqrt{1 + \left(\frac{e^{2x}}{4} - \frac{1}{2} + \frac{e^{-2x}}{4}\right)}\,dx$$

$$= \int_0^{\ln 2} \sqrt{\frac{e^{2x}}{4} + \frac{1}{2} + \frac{e^{-2x}}{4}}\,dx = \int_0^{\ln 2} \sqrt{\left(\frac{e^x}{2} + \frac{e^{-x}}{2}\right)^2}\,dx$$

$$= \int_0^{\ln 2} \left(\frac{e^x}{2} + \frac{e^{-x}}{2}\right)dx = \left[\frac{e^x}{2} - \frac{e^{-x}}{2}\right]_0^{\ln 2}$$

$$= \left[\frac{e^{\ln 2}}{2} - \frac{1}{2e^{\ln 2}}\right] - \left[\frac{1}{2} - \frac{1}{2}\right]$$

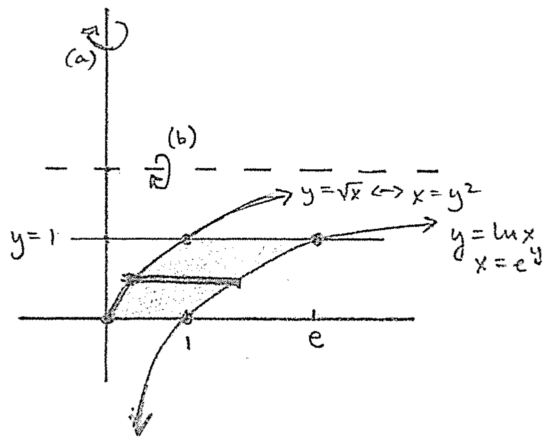
$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

[8 pt] 7. Let $\mathcal{R}$ be the region bounded by $y = \sqrt{x}$, $y = \ln x$, $y = 1$, and the x-axis.

(a) State, but do not evaluate, the integral that gives the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

(b) State, but do not evaluate, the integral that gives the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = 2$.



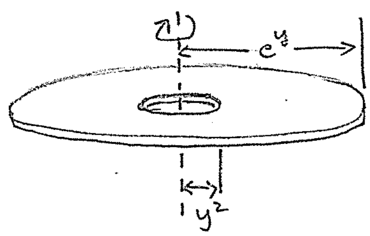
Points of intersection

$\sqrt{x} = 1 \Rightarrow x = 1$

$\ln x = 1 \Rightarrow x = e$

$\ln x = 0 \Rightarrow x = 1$

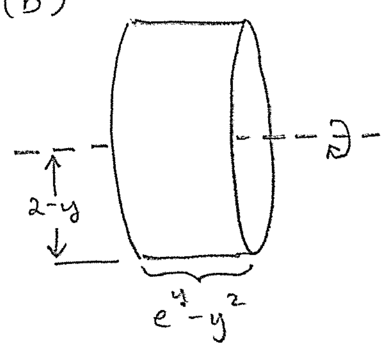
(a)



$$V = \int_0^1 \pi\left((e^y)^2 - (y^2)^2\right) dy$$

$$= \int_0^1 \pi\left(e^{2y} - y^4\right) dy$$

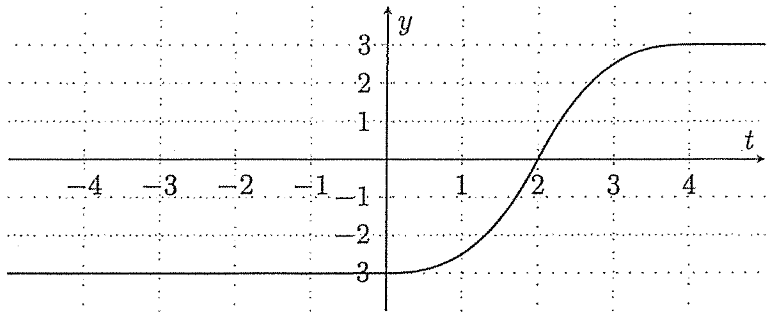
(b)



$$V = \int_0^1 2\pi(2-y)(e^y - y^2)\, dy$$

$$= \int_0^1 2\pi\left(2e^y - ye^y - 2y^2 + y^3\right) dy$$

8. In all parts of this question, $f(t)$ is the function shown in the graph below.



[2 pt] (a) ***Estimate*** the value of $\int_{-2}^{4} f(t)\,dt$ by evaluating a Riemann sum with right endpoints and $n = 3$ (with all subintervals having equal widths).

[2 pt] (b) If $\int_0^2 f(t)\,dt = -\frac{13}{3}$, what is $\int_{-2}^{2} f(t)\,dt$?

[1 pt] (c) If $g(x) = \int_0^x f(t)\,dt$, what is the value of $g'(-2)$?

(a) $\Delta x = \frac{6}{3} = 2 \implies R_n = f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2$

$= (-3)(2) + (0)(2) + (3)(2)$

$= 0$

(b) $\int_{-2}^{2} f(t)dt = \int_{-2}^{0} f(t)dt + \int_0^2 f(t)dt = -6 - \frac{13}{3} = -\frac{31}{3}$

(c) $g'(x) = f(x) \implies g'(-2) = f(-2) = -3.$

[3 pt] 9. Suppose that $g(x) = \int_0^x e^{-t^3}\,dt$. Find $\lim_{x\to 0^+} \frac{\int_0^x g(t)\,dt}{g(x)}$.

$\int_0^0 g(t)dt = 0$ and $g(0) = \int_0^0 e^{-t^3}dt = 0$; so the limit is $\frac{0}{0}$ form.

$$\lim_{x\to 0^+} \frac{\int_0^x g(t)\,dt}{g(x)} = \lim_{x\to 0^+} \frac{g(x)}{g'(x)} = \lim_{x\to 0^+} \frac{g(x)}{e^{-x^3}} = \frac{g(0)}{e^0} = \frac{0}{1} = 0.$$

[4 pt] 10. Evaluate the limit by expressing it as a definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{3+(1+\frac{2i}{n})^2}\frac{2}{n}$$

$$\left.\begin{array}{l} a+i\Delta x = 1+\frac{2i}{n} \\ \Delta x = \frac{2}{n} \end{array}\right\} \Rightarrow a=1,\ b=3,\ f(x)=\frac{1}{3+x^2}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{3+(1+\frac{2i}{n})^2}\cdot\frac{2}{n} = \int_1^3 \frac{1}{3+x^2}\,dx$$

$$= \left[\frac{1}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)\right]_1^3$$

$$= \frac{1}{\sqrt{3}}\left(\arctan\sqrt{3} - \arctan\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{6}\right) = \frac{\pi}{6\sqrt{3}}.$$

[3 pt] 11. Suppose that $\int_0^a \frac{x(1-e^x)}{e^x+1}\,dx = -1$. Find $\int_{-a}^a \frac{x(1-e^x)}{e^x+1}\,dx$ without finding the antiderivative of the integrand. Show your work.

Determine if $f(x)=\frac{x(1-e^x)}{e^x+1}$ is odd, even, or neither.

$$f(-x) = \frac{(-x)(1-e^{-x})}{e^{-x}+1} = \frac{-x(1-\frac{1}{e^x})}{\frac{1}{e^x}+1}\cdot\frac{e^x}{e^x} = \frac{-x(e^x-1)}{1+e^x} = \frac{x(1-e^x)}{e^x+1} = f(x)$$

$\therefore f(x)$ is even

$$\Rightarrow \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx = 2(-1) = -2$$

[1 pt] 12. (a) Show that $0 < \frac{(n!)^2}{(2n)!} < 1$ when $n \geq 1$.

[3 pt] (b) Use part (a) (whether or not you have shown it) to determine if the following sequence converges or diverges. If it converges, find its limit.

$$a_n = \frac{(n!)^2}{(2n+1)!}, \quad n \geq 1.$$

(a) $\frac{(n!)^2}{(2n)!} > 0$ since factorials are positive

$$\frac{(n!)^2}{(2n)!} = \frac{(n!)(1)(2)\cdots(n)}{n!(n+1)(n+2)\cdots(2n)} = \frac{1}{n+1}\cdot\frac{2}{n+2}\cdot\ \cdots\ \cdot\frac{n}{2n} < 1$$

(b)

$$0 < \frac{(n!)^2}{(2n)!} < 1$$

$$\Rightarrow 0 < \frac{(n!)^2}{(2n)!\,(2n+1)} < \frac{1}{2n+1}$$

$$\Rightarrow 0 < a_n < \frac{1}{2n+1} \to 0$$

$\therefore$ a_n converges to 0 by the Squeeze Theorem.

13. Determine if each of the following series converges or diverges. If it converges, find its sum.

[4 pt] (a) $\pi - \frac{2\pi}{e} + \frac{4\pi}{e^2} - \frac{8\pi}{e^3} + \frac{16\pi}{e^4} - \frac{32\pi}{e^5} - \ldots$

Geometric with $a = \pi$, $r = -\frac{2}{e}$

Since $|r| = \frac{2}{e} < 1$, the series converges to

$$S = \frac{a}{1-r} = \frac{\pi}{1+2/e} = \frac{\pi e}{e+2}$$

[2 pt] (b) $\sum_{n=1}^{\infty} a_n$ where the n^{th} partial sum is $s_n = \frac{3-n}{1+2n}$

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty}\left(\frac{3-n}{1+2n}\right) = -\frac{1}{2}$$

$\therefore$ the series converges to $-\frac{1}{2}$.

14. Determine if each of the following series is absolutely convergent, conditionally convergent, or divergent. Clearly state which test you are using, verify that this test is valid, and give justification for your conclusion.

[4 pt] (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$

Integral Test with $f(x) = \dfrac{1}{x\sqrt[3]{\ln x}}$, $[2, \infty)$

- $f(x) \geq 0$? Yes, x & $(\ln x)^{1/3}$ both positive
- $f(x)$ cts? Yes, $x \neq 0, 1$
- $f(x)$ dec? Yes, $x\sqrt[3]{\ln x}$ is increasing

$$\int_2^{\infty} \frac{1}{x\sqrt[3]{\ln x}}\,dx = \lim_{t\to\infty} \int_2^t (\ln x)^{-1/3} \cdot \frac{1}{x}\,dx$$

$u = \ln x$ $\quad x = 2 \Rightarrow u = \ln 2$

$du = \frac{1}{x}dx$ $\quad x = t \Rightarrow u = \ln t$

$$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} u^{-1/3}\,du$$

$$= \lim_{t\to\infty} \left[\frac{3}{2}u^{2/3}\right]_{\ln 2}^{\ln t}$$

$$= \lim_{t\to\infty} \left[\frac{3}{2}(\ln t)^{2/3} - \frac{3}{2}(\ln 2)^{2/3}\right] = \infty$$

Since $\displaystyle\int_2^{\infty} \frac{1}{x\sqrt[3]{\ln x}}\,dx$ diverges, so does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$.

[4 pt] (b) $\displaystyle\sum_{n=6}^{\infty} \left(1 - \frac{5}{n}\right)^{n^2}$

Root test (no conditions to satisfy):

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(1-\frac{5}{n}\right)^{n^2}\right|} = \lim_{n\to\infty} \left(1 - \frac{5}{n}\right)^n = L \quad (1^{\infty})$$

$$\Rightarrow \ln L = \lim_{n\to\infty} \ln\left(\left(1-\frac{5}{n}\right)^n\right)$$

$$= \lim_{n\to\infty} n\ln\left(1-\frac{5}{n}\right)$$

$$= \lim_{n\to\infty} \frac{\ln\left(1-\frac{5}{n}\right)}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{(1-5/n)} \cdot \frac{5}{n^2}}{\frac{-1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{1}{1-\frac{5}{n}} \cdot \frac{5}{\cancel{n^2}} \cdot \frac{\cancel{n^2}}{-1}$$

$$= (1)(5)(-1) = -5$$

$\ln L = -5$

$L = e^{-5} = \dfrac{1}{e^5} < 1$

$\therefore$ the series is absolutely convergent.

[4 pt] (c) $\displaystyle\sum_{n=2}^{\infty}(-1)^{n+1}\sqrt{\frac{1}{n^2-2}}$

Alternating series

$$a_n = (-1)^{n+1}\frac{1}{\sqrt{n^2-2}}, \qquad b_n = |a_n| = \frac{1}{\sqrt{n^2-2}}$$

① $b_n \to 0$?

$$\lim_{n\to\infty}\frac{1}{\sqrt{n^2-2}} = 0 \quad ✓$$

② b_n decreasing? $(n+1)^2-2 > n^2-2$

$$\Rightarrow \frac{1}{\sqrt{(n+1)^2-2}} < \frac{1}{\sqrt{n^2-2}} \quad \therefore b_{n+1} < b_n \quad ✓$$

$\therefore \sum a_n$ converges.

Now consider $\displaystyle\sum|a_n| = \sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2-2}}$ (positive terms $\rightarrow$ comparison test)

$$\frac{1}{\sqrt{n^2-2}} > \frac{1}{\sqrt{n^2}} = \frac{1}{n}$$

Since $\sum \frac{1}{n}$ diverges, so does $\displaystyle\sum_{n=2}^{\infty}\frac{1}{\sqrt{n^2-2}}$ by the Direct Comparison Test.

Since $\sum a_n$ converges but $\sum|a_n|$ diverges, $\sum a_n$ is conditionally convergent.

[4 pt] 15. (a) Find the interval of convergence of the power series $\sum_{n=0}^{\infty}(-1)^n(n+1)(x-3)^n$.

[4 pt] (b) Show that the Taylor series for $f(x) = \dfrac{1}{(x-2)^2}$ centered at $a = 3$ is $\sum_{n=0}^{\infty}(-1)^n(n+1)(x-3)^n$.

[1 pt] (c) Use (a) and (b) to show that $\sum_{n=0}^{\infty}\dfrac{n+1}{2^n} = 4$. Hint: $x = \dfrac{5}{2}$ should lie in the interval of convergence you found in (a).

(a) Ratio Test:

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+2)(x-3)^{n+1}}{(-1)^n(n+1)(x-3)^n}\right| = \lim_{n\to\infty}\frac{n+2}{n+1}\cdot|x-3| = |x-3|$$

The series converges when $|x-3|<1 \iff -1<x-3<1$
$\iff 2<x<4$

At $x=2 \Rightarrow \sum_{n=0}^{\infty}(-1)^n(n+1)(-1)^n = \sum_{n=0}^{\infty}(n+1)$

At $x=4 \Rightarrow \sum_{n=0}^{\infty}(-1)^n(n+1)(1)^n = \sum_{n=0}^{\infty}(-1)^n(n+1)$

Each of these diverges by the n^{th} term test, so the interval of convergence is $2<x<4$.

(b) $f^{(0)}(x) = (x-2)^{-2}$

$f^{(1)}(x) = -2(x-2)^{-3}$

$f^{(2)}(x) = 2\cdot 3\,(x-2)^{-4}$

$f^{(3)}(x) = -2\cdot 3\cdot 4\,(x-2)^{-5}$

$f^{(n)}(x) = (-1)^n(n+1)!\,(x-2)^{-(n+2)}$

$f^{(n)}(3) = (-1)^n(n+1)!\,(1)$
$= (-1)^n(n+1)!$

The Taylor series is

$$\sum_{n=0}^{\infty}\frac{f^{(n)}(3)}{n!}(x-3)^n$$
$$= \sum_{n=0}^{\infty}\frac{(-1)^n(n+1)!}{n!}(x-3)^n$$
$$= \sum_{n=0}^{\infty}(-1)^n(n+1)(x-3)^n$$

(c) $$f\left(\tfrac{5}{2}\right) = \sum_{n=0}^{\infty}(-1)^n(n+1)\left(\tfrac{5}{2}-3\right)^n$$

$$\frac{1}{\left(\frac{5}{2}-2\right)^2} = \sum_{n=0}^{\infty}(-1)^n(n+1)\left(\tfrac{-1}{2}\right)^n$$

$$\frac{1}{(1/2)^2} = \sum_{n=0}^{\infty}\frac{(-1)^n(n+1)(-1)^n}{2^n} \Rightarrow 4 = \sum_{n=0}^{\infty}\frac{n+1}{2^n}$$