DAWSON COLLEGE
Mathematics Department

FINAL EXAMINATION

Calculus II (201-NYB-05)-Open
Winter 2022

Instructor:
Noushin Sabetghadam

Student Name: ______________________________

Student ID. #: ______________________________

Instructions:

- Print your name and ID number in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly and indicate your final answer.
- Only calculators Sharp EL 531.X/ XG/XT are permitted.
- The last page is a copy of the formula sheet. Note that the examination booklet must be returned intact.

This examination consists of 13 questions. Please ensure that you have a complete examination booklet before starting.

Q1/6	Q2/6	Q3/6	Q4/20	Q5/6	Q6/6	Q7/8	Q8/6	Q9/6	Q10/6	Q11/6	Q12/12	Q13/6	Sum

(1) **(6 marks)** Use ONLY the Riemann Sum technique to evaluate the integral $\int_{-1}^{3} (2x^2 - 3x)dx$.

(2) **(6 marks)** Use the Fundamental Theorem of Calculus to find the derivative of the given function.

$$g(x) = \int_{\cos x}^{3x^2} \frac{\sqrt{t + \sqrt{t}}}{1 - t}\, dt$$

(3) **(6 marks)** Find the average value of $f(x) = \frac{1}{x^2}\cos(\frac{2\pi}{x})$ over the interval $[1, 4]$.

(4) Find the following integrals.

(a) (6 marks) $\int \sin^4(3x)\cos^5(3x)dx$

(b) (6 marks) $\displaystyle\int \frac{dx}{(16-x^2)^{\frac{3}{2}}}$

(c) (8 marks) $\displaystyle\int \frac{3x^2-3x+20}{x(x^2+4)}\,dx$

(5) **(6 marks)** Find the volume of the solid obtained when the region bounded by $y = \sqrt{x-1}$ and $y = x - 1$ is rotated about the x-axis.

(6) **(6 marks)** Compute the following improper integral or show it diverges.

$$\int_{-\infty}^{2} xe^{x}\,dx$$

(7) **(8 marks)** Nintendo decides to sell a new gaming system and does market and internal research to determine the supply and demand curves:

$$D(x) = 470 + \frac{330,000}{x + 1000}, \qquad S(x) = 510 - \frac{110,000}{x + 1000}$$

Where $D(x)$ is the demand in dollars/unit and $S(x)$ is the supply in dollars/unit and x is the units of the gaming system sold/produced. Find:

(a) The equilibrium market quantity and price.

(b) Find the Producers' surplus. (No decimal is needed)

(8) **(6 marks)** Find the general solution of the differential equation $(y^2+1)e^x - yy' = 0$.

(9) **(6 marks)** Determine whether the following sequence is convergent or divergent. Find its limit if it converges.

$$a_n = \left(\frac{n+1}{n}\right)^{3n}$$

(10) **(6 marks)** Determine whether the series converges or diverges. If it converges, find its sum.

$$\sum_{n=0}^{\infty} \frac{5^{(n-1)} - 2^{(2n)}}{10^{(n+1)}}$$

(11) **(6 marks)** Find the Taylor series representation of $f(x) = \dfrac{2}{1-x}$ centred at $a = 2$. Then find the radius of convergence.

(12) Determine whether the given series are convergent or divergent. Justify your answer.

(a) (4 marks) $\displaystyle\sum_{n=0}^{\infty} \frac{1+5n}{\sqrt{4n^2+2n+1}}$

(b) (4 marks) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$

(c) (4 marks) $\displaystyle\sum_{n=0}^{\infty} \frac{4n}{3n^3+n}$

(13) **(6 marks)** Use the Simpson's rule with $n = 4$ to approximate the value of the definite integral. Round the final answer up to five decimal places.

$$\int_1^3 \frac{e^x}{\sqrt{x+1}}\, dx$$

Dawson College
Mathematics Department
Calculus II, 201-NYB-05(Commerce/Open)
Information Sheet

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\int_a^b f(x)dx \approx \frac{\triangle x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$\int_a^b f(x)dx \approx \frac{\triangle x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

$$\mathbf{CS} = \int_0^{\bar{x}} D(x)\ dx - \bar{p}\bar{x}$$

$$\mathbf{PS} = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\ dx$$

$$\mathbf{P_n(x)} = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$$\mathbf{P_n(x)} = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Final Answers

(1) $\frac{20}{3}$

(2) $g'(x) = \frac{\sqrt{3x^2 + \sqrt{3x^2}}}{1 - 3x^2} \cdot 6x + \frac{\sqrt{\cos x + \sqrt{\cos x}}}{1 - \cos x} \cdot \sin x$

(3) $f_{ave} = \frac{-1}{6\pi}$

(4) (a) $I = \frac{\sin^5(3x)}{15} - \frac{2\sin^7(3x)}{21} + \frac{\sin^9(3x)}{27} + C$

(b) $I = \frac{x}{64\sqrt{1 - \frac{x^2}{16}}}$

(c) $I = 5\ln|x| - \ln(x^2 + 4) - \frac{3}{2}\arctan\frac{x}{2} + C$

(5) $V = \frac{1}{6}\pi$

(6) $I = e^2$, Convergent

(7) (a) $x = 10000,\ p = 500\$$

(b) $163,768\$$

(8) $y = \pm\sqrt{Ce^{2e^x} - 1}$

(9) It converges to 3.

(10) It converges to $\frac{-19}{150}$.

(11) $\sum_{0}^{\infty} 2(-1)^{n+1}(x-2)^n$, the radius of convergence is 1.

(12) (a) Divergent using the test for divergence.

(b) Divergent using the integral test.

(c) Convergent using limit comparison test.

(13) 9.64703