DAWSON COLLEGE
Mathematics Department

FINAL EXAMINATION

Calculus II-Commerce/Open- 201-NYB-05
Winter 2019

Instructor: Noushin Sabetghadam

Student Name: ______________________________

Student ID. #: ______________________________

Instructions:

- Print your name and ID number in the provided space.
- For the multiple choice questions 1-15, choose only one letter for the answer and write it down in the table provided.
- Solve the problems 16-25 in the space provided for each question and show all your work clearly and indicate your final answer.
- Only calculators Sharp EL 531.X/ XG/XT are permitted.
- This examination booklet must be returned intact.

This examination consists of 25 questions. Please ensure that you have a complete examination booklet before starting.

Write only **the letter of the answer** that you choose for the first 15 multiple choice questions in the following table. No mark given if you choose more than one letter.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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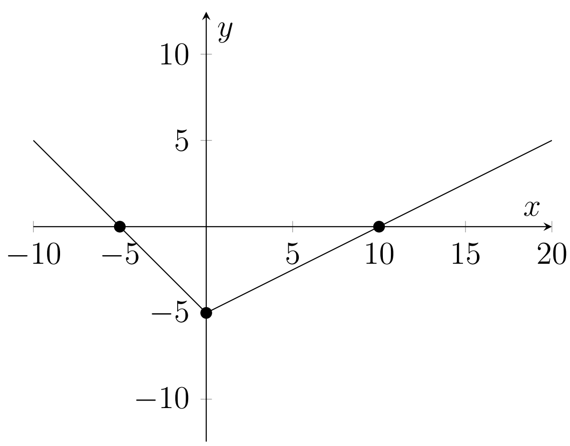
(1) **(2 marks)** Suppose $f'(x) = 3e^x + 2x$ and $f(0) = 2$. What is $f(1)$?

(a) $3e$
(b) $3e + 1$
(c) $3e - 1$
(d) $3e + 2$
(e) $3e - 2$

(2) **(2 marks)** The graph of $y = f(x)$ is given, what is the value of $\int_{-5}^{10} f(x)dx$?

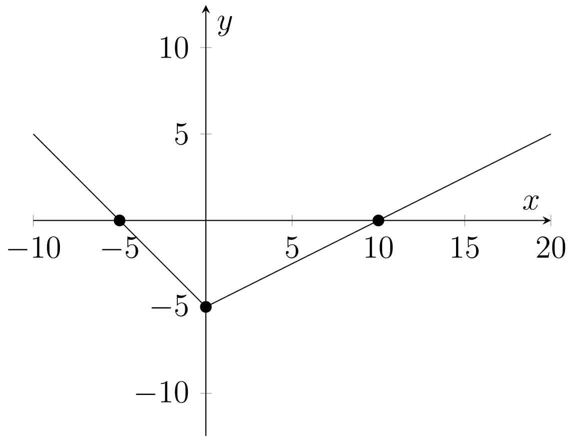


(a) 12.5
(b) −12.5
(c) 37.5
(d) −37.5
(e) −75

(3) **(2 marks)** Suppose $\int_{-1}^{4} 2f(x)\,dx = 3$ and $\int_{2}^{4} f(x)\,dx = -3$. Find $\int_{-1}^{2}(f(x) + x)\,dx$.

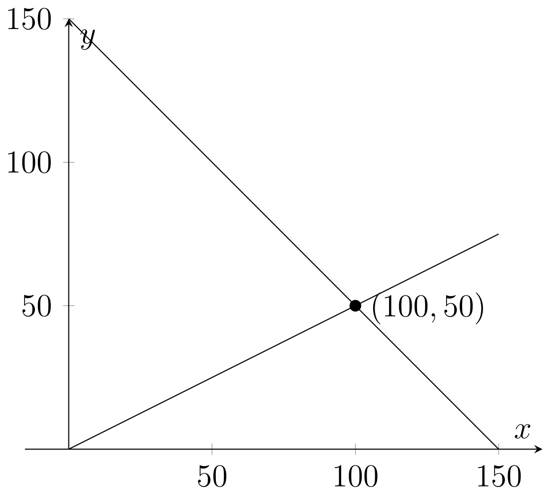
(a) 6
(b) $\frac{3}{2}$
(c) $\frac{-1}{2}$
(d) $\frac{9}{2}$
(e) $\frac{15}{2}$

(4) **(2 marks)** Let $g(x) = \int_0^x f(t)dt$ where the graph of the function $y = f(x)$ is given as below. What is the value of $g'(-5)$?



(a) 12.5
(b) −12.5
(c) 25
(d) −5
(e) 0

(5) **(2 marks)** The graph of the demand and the supply functions are given as below. Find the **producers' surplus** if the market price is set at the equilibrium $(100, 50)$.



(a) \$15000
(b) \$10000
(c) \$7500
(d) \$5000
(e) \$2500

(6) **(2 marks)** The value of $\int_1^e \ln x \, dx$ is:

(a) $e - 1$
(b) $1 - e$
(c) 1
(d) e
(e) $e - 2$

(7) **(2 marks)** The value of $\int_1^2 \frac{1}{x(x+1)}\,dx$ is:

(a) $\ln 2 - \ln 3$
(b) $\ln 4 - \ln 3$
(c) $1/3$
(d) $-1/3$
(e) $\ln 2 - 2\ln 3$

(8) **(2 marks)** Find the $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$ if exists.

(a) 1
(b) e^2
(c) $e+2$
(d) e
(e) ∞

(9) **(2 marks)** Determine whether $\int_{-\infty}^{0} \frac{2}{1+x^2}\,dx$ is convergent or divergent. If it converges, evaluate the integral.

(a) divergent
(b) convergent, $-\pi$
(c) convergent, $-\pi/2$
(d) convergent, $\pi/2$
(e) convergent, π

(10) **(2 marks)** A region is bounded above (and left) by the graph of $f(x) = x^2$, below by the $x-$axis and on the right by $x = 1$. The solid object obtained by rotating this area about the y-axis is obtained by:

(a) $\int_0^1 \pi(x^4)\,dx$
(b) $\int_0^1 \pi(x^4-1)\,dx$
(c) $\int_0^1 \pi(y^4)\,dy$
(d) $\int_0^1 \pi(1-y)\,dy$
(e) $\int_0^1 \pi y\,dy$

(11) **(2 marks)** Determine the convergence or divergence of a sequence $\{a_n\}$ with the given general term and find the limit if it is convergent.

$$a_n = \frac{2^n - 3^{n+2}}{3^n}$$

(a) divergent
(b) convergent, -9
(c) convergent, 9
(d) convergent, 0
(e) convergent, $-1/3$

(12) **(2 marks)** Determine the convergence or divergence of the series $\sum_{n=2}^{\infty} \left(\frac{1}{\ln(n+1)} - \frac{1}{\ln(n+2)} \right)$ and find the limit if it is convergent.

(a) divergent
(b) convergent, 1
(c) convergent, 0
(d) convergent, $\frac{1}{\ln 2}$
(e) convergent, $\frac{1}{\ln 3}$

(13) **(2 marks)** Find the sum of the series $\sum_{n=0}^{\infty} a_n$ if exists, where the n-th partial sum $S_n = \frac{2-n}{1+3n}$.

(a) 2
(b) 2/3
(c) $-1/3$
(d) -1
(e) It is divergent.

(14) **(2 marks)** Determine whether the infinite geometric series

$$5 - 0.02 + (0.02)^2 - (0.02)^3 + (0.02)^4 - (0.02)^5 + \cdots$$

converges. If the series converges, determine the limit.

(a) Converges; $\frac{244}{49}$

(b) Converges; $\frac{-244}{49}$

(c) Converges; $\frac{256}{51}$

(d) Diverges,

(e) Converges; $\frac{254}{51}$

(15) **(2 marks)** Find the first 4 non-zero terms of the n-th Taylor Polynomial for $f(x) = e^{-x^2}$ centered at 0.

(a) $1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$

(b) $1 - x + x^2 - x^3$

(c) $1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6$

(d) $1 - x^2 + x^4 - x^6$

(e) $1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6$

Solve the problems 16-25 in the space provided for each question and show all your work clearly and indicate your final answer.

(16) **(5 marks)** Use ONLY the Riemann Sum technique to evaluate the integral $\int_{-1}^{2} (2x - 3x^2)dx$.

(17) **(5 marks)** Find the average value of $f(x) = \sin^3(2x)\cos^2(2x)$ over the interval $[0, \frac{\pi}{4}]$.

(18) **(20 marks)** Find the integrals.

(a) $\displaystyle\int_{-4}^{0} \frac{x^3}{\sqrt{9+x^2}}\, dx$

(b) $\displaystyle\int x^2 \cos x \, dx$

(c) $\displaystyle\int \frac{x^2 - x - 7}{(x-2)(x^2+1)}\,dx$

(d) $\displaystyle\int \frac{1}{(4-t^2)^{3/2}}\,dt$

(19) **(7 marks)** Consider the region enclosed by the curve $y = -x^2 + 4x$ and the line $y = x$.

(a) Evaluate the area of the region.

(b) Set up only, **but do not evaluate the integral** for the volume of the solid when the given region is rotated about the horizontal line $y = 5$.

(20) **(5 marks)** Evaluate the limit: $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

(21) **(5 marks)** Determine whether the improper integral is divergent or convergent. Evaluate it if it is convergent. $\displaystyle\int_3^7 \frac{2}{\sqrt{x-3}}\,dx$

(22) **(5 marks)** Find the exact length of the curve $y = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$ where $(\ln 2 \leq x \leq 1)$.

(23) **(5 marks)** Solve the differential equation $y' = xye^{(x^2)}$ with the initial condition $y(0) = e$.

(24) **(5 marks)** Find the **radius** and the **interval** of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-2)^n(x-2)^n}{2n+1}$

(25) **(8 marks)** Determine whether the series is absolutely convergent, conditionally convergent, or divergent. State which test you are using for each problem.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n \sqrt{n}}{(n+1)!}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(2n)}{\ln(n^2+1)}$