DAWSON COLLEGE
MATHEMATICS DEPARTMENT
Calculus II(SCIENCE)

I confirm that I have read and understood the College's Academic Integrity Document and will adhere to the principles of academic integrity while writing this exam.

201-NYB-05 S01-S14
Winter 2023
Final Examination
May 23rd, 2023
Time Limit: 3 hours

Name: ______________________

ID#: ______________________

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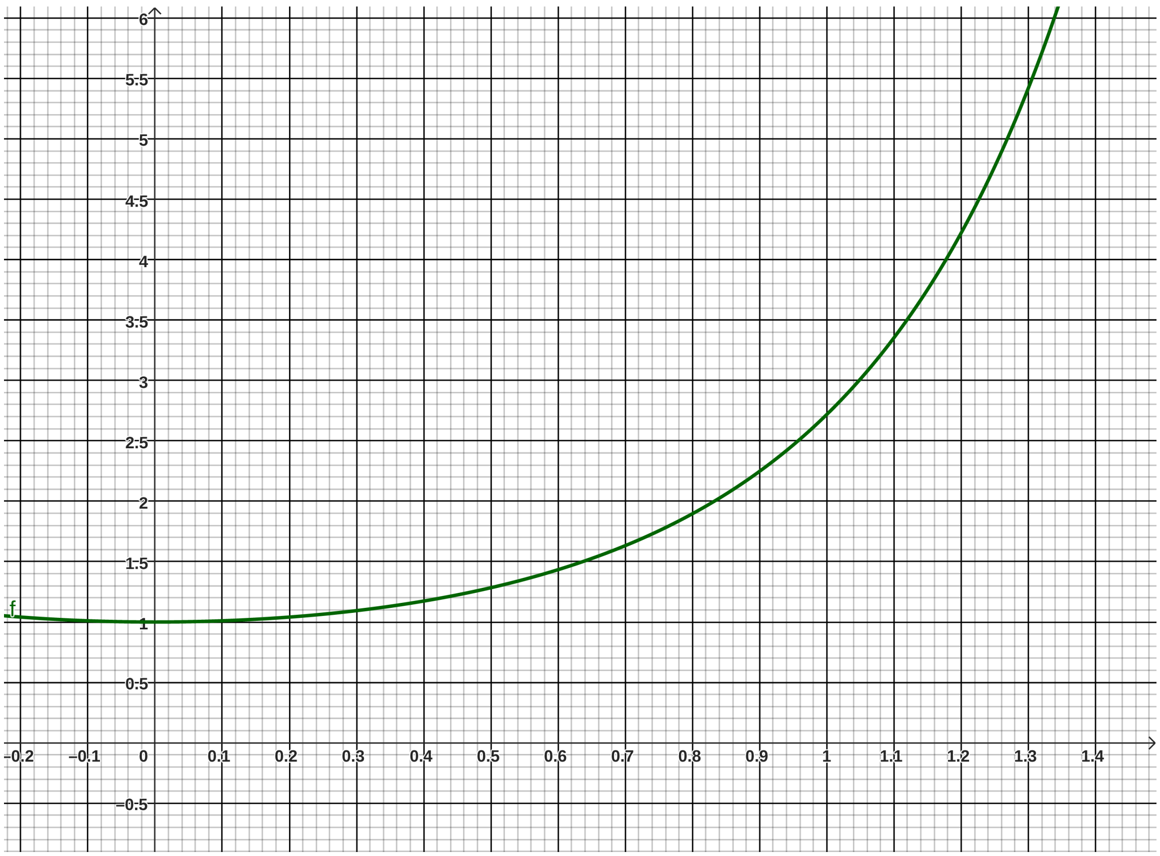
- This exam contains 6 pages (including this cover page) and 19 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your solutions in a legible and orderly manner.
- Good luck!

Question	Points	Score
1	5	
2	9	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
13	3	
14	5	
15	10	
16	5	
17	5	
18	4	
19	4	
Total:	100	

1. Given the graph of $f(x) = e^{x^2}$.



   (a) (3 marks) Find an approximation of the definite integral of $f(x) = e^{x^2}$ on the interval $[1/5,\ 4/5]$, using the right end point as sample points and 3 approximating rectangles.

   Answer: ≈ 0.9

   (b) (1 mark) Sketch the approximating rectangles.

   (c) (1 mark) Is the estimate an overestimate, underestimate or neither? Justify.

   Answer: overestimate

2. Consider the region in the first quadrant $\mathscr{R}$ bounded by $y = e^{x^2}$, $x = 1$ and $y = 1$. *See the graph of $y = e^{x^2}$ in problem 1.*

   (a) (5 marks) Use the cylindrical shell method to find the volume of the solid obtained by revolving the region $\mathscr{R}$ about the y-axis.

   Answer: $\pi(e-2)$

   (b) (4 marks) Using discs or washers, set up, **but do not evaluate**, the integral required to find the volume of the solid obtained by revolving the region $\mathscr{R}$ about the line $y = \frac{1}{2}$.

   Answer:

$$\int_0^1 \pi\left(\left(e^{x^2} - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)\, dx$$

3. (5 marks) Consider the region(s) $\mathscr{R}$ bounded by the following: $y = x^2 - x - 1$, $y - x = 2$, $x = 4$. Set up, **but do not evaluate**, an integral or integrals that represents the area of the region.

Answer: $\int_{-1}^{3} x + 2 - (x^2 - x - 1)\,dx + \int_{3}^{4} x^2 - x - 1 - (x+2)\,dx$

4. (5 marks) Find the exact length of the curve $y(x) = \displaystyle\int_3^x \sqrt{t^2 + 2t}\,dt$ on $[1,4]$.

Answer: $\frac{21}{2}$

5. (5 marks) Evaluate the following integral

$$\int_{-\pi/12}^{\pi/12} \left(\frac{(x^2+1)\sin x}{x^4+1} + \cos^2 2x \right) dx$$

Answer: $\frac{\pi}{12} + \frac{\sqrt{3}}{8}$

6. (5 marks) Find the average value of $f(x) = \frac{x^8}{\sqrt{x^3+1}}$ on the interval $[0,2]$.

Answer: $\frac{496}{45}$

7. (5 marks) Evaluate the following integral

$$\int_0^{1/3} \frac{1}{(9x^2+1)^{5/2}}\,dx$$

Answer: $\frac{5}{18\sqrt{2}}$

8. (5 marks) Evaluate the following integral

$$\int \frac{7x^2 + x + 1}{x^3 + x^2 + x}\,dx$$

Answer: $\ln|x| + 3\ln|x^2+x+1| - \frac{6}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C$

9. (5 marks) Evaluate the following integral

$$\int \frac{e^{4x} + e^{2x} + e^x \sec(\ln(e^x+1))}{e^x+1}\,dx$$

Answer: $e^{3x} - e^{2x} + 2e^x - \ln|e^x + 1| + \ln|\sec(\ln(e^x + 1)) + \tan(\ln(e^x + 1))| + C$

10. (5 marks) Evaluate the improper integral or show it diverges

$$\int_1^\infty \frac{\ln x}{x^{3/2}}\, dx$$

Answer: 4

11. (5 marks) Evaluate the improper integral or show it diverges

$$\int_0^{3\pi/2} \frac{\cos\theta}{\sqrt{1 - \sin\theta}}\, d\theta$$

Answer: $2 - 2\sqrt{2}$

12. (5 marks) Given that

1. $f(x)$ and $g(x)$ are continuous functions on $\mathbb{R}$,
2. $g(x) \geq f(x)$ on $[0,\ 3]$ and that the area between $f(x)$ and $g(x)$ on $[0,\ 3]$ is 7,
3. $\displaystyle\int_0^3 \Big(f(x) - 7g(x)\Big)\, dx = 3.$

Find $\displaystyle\int_3^0 g(x)\, dx$.

Answer: $\frac{5}{3}$

13. (3 marks) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{1 + \sqrt{2},\ 3,\ 1 + \sqrt{8},\ 5,\ 1 + \sqrt{32},\ 9,\ 1 + \sqrt{128},\ \ldots\right\}$$

Answer: $a_n = 1 + \sqrt{2^n}$

14. (5 marks) Determine whether the sequence converges or diverges. If it converges, find the limit. *Hint: Evaluate the limit as a definite integral.*

$$a_n = \sum_{i=1}^{n} \frac{\tan^2\left(\frac{i}{n}\right)}{n}$$

Answer: $\tan(1) - 1$

15. Determine whether the series are convergent or divergent. Justify your answer.

   (a) (5 marks) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 3^n + n}{\sqrt{n} + n^3 4^n}$

   Answer: converges

   (b) (5 marks) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n + e^{-n}}$

   Answer: diverges

16. Given the series $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{\arctan(n)}{\arctan(n+1)}\right)$

   (a) (3 marks) Find an expression for the n$^{\text{th}}$ partial sum.

   Answer: $S_n = \ln\left(\frac{\pi}{4}\right) - \ln\left(\arctan\left(n+1\right)\right)$

   (b) (2 marks) Use part a. to determine whether the series is conververgent or divergent. If it is convergent, find its sum.

   Answer: $\ln\left(\frac{1}{2}\right)$

17. (5 marks) Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} a_n(x)$ where $a_n(x) = \dfrac{(3x-1)^n}{2^n\sqrt{n}}$ given that $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \frac{|3x-1|}{2}$.

Answer: $R = \frac{2}{3}$, $(-\frac{1}{3}, 1]$

18. (4 marks) Find the Taylor series of $f(x) = \frac{1}{\sqrt{12+7x}}$ about $x = 3$ given that

$$\begin{aligned}
f(x) &= \frac{1}{\sqrt{12+7x}} \\
f'(x) &= \frac{-7}{2(12+7x)^{3/2}} \\
f''(x) &= \frac{3\cdot 7^2}{2^2(12+7x)^{5/2}} \\
f'''(x) &= \frac{-3\cdot 5\cdot 7^3}{2^3(12+7x)^{7/2}} \\
f^{(4)}(x) &= \frac{3\cdot 5\cdot 7\cdot 7^4}{2^4(12+7x)^{9/2}} \\
f^{(5)}(x) &= \frac{-3\cdot 5\cdot 7\cdot 9\cdot 7^5}{2^5(12+7x)^{11/2}}
\end{aligned}$$

Answer:

$$\frac{1}{\sqrt{33}} + \sum_{n=1}^{\infty} \frac{(-1)^n 7^n 1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2^n n!(\sqrt{33})^{2n+1}}(x-3)^n$$

19. (4 marks) Given that the series $\sum_{n=1}^{\infty} a_n$ is convergent. Determine whether the series $\sum_{n=1}^{\infty} \arccos(a_n)$ is convergent.

Answer: divergent