DAWSON COLLEGE
Mathematics Department
Final Examination
Linear Algebra
201-NYC –05 (Computer Science)
Winter 2019

1. a) (5 marks) Use Gauss-Jordan elimination to find the general solution of the system.
   b) (1 mark) Find the particular solution of the system in which $x_3 = 0$ and $x_2 = 4$.

$$\begin{aligned} x_1 - x_2 - 2x_3 + 6x_4 &= 3 \\ -2x_1 - x_2 + 7x_3 + 3x_4 &= -3 \\ 3x_1 - 2x_2 - 7x_3 + 13x_4 &= 8 \end{aligned}$$

2. (5+3 marks) Given the system of linear equations $\begin{cases} 2x + 2y - z = 5 \\ x - y - 2z = -3 \\ 2x + y - z = 3 \end{cases}$.

a) Use the adjoint matrix to find the inverse of the coefficient matrix.
b) Use the inverse of the coefficient matrix to solve the system.

3. (4 marks) Use Cramer's rule to solve the system $\begin{cases} 2x + 2y - z = 5 \\ x - y - 2z = -3 \\ 2x + y - z = 3 \end{cases}$ for y only.

4. (3 marks) If A, B and C are $n \times n$ invertible matrices then simplify the following expression $\left(2AC^T A^2\right)^{-1} \cdot \left(A^{-1} C A^T\right)^T \cdot \left(8B^0 A\right)^T$.

5. (8 marks) Given the following matrices $A = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -1 \\ 0 & -2 \\ 1 & 2 \end{bmatrix}$.

   a) Calculate $tr\left(C^T C - A^2 + 6B^{-1}\right)$. b) Solve for X : $\left(X^T - 4I^3\right)^{-1} = A$,

6. (3+1 marks) If $A = \begin{bmatrix} 3 & -2 & 4 \\ -1 & 5 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 & -3 \\ 1 & 8 & -2 \end{bmatrix}$, then
   a) Find elementary matrices E_1 and E_2 such that $E_2 E_1 A = B$.
   b) Does the homogeneous system $AX = 0$ have a non-trivial solution?

7. (4 marks) For which values of k does the following system $\begin{cases} x + 5y - 3z = 2 \\ -2x - 9y + 7z = -3 \\ -x - 5y + \left(k^2 - 6\right) z = k + 1 \end{cases}$ have
   1) exactly one solution, 2) infinitely many solutions, 3) no solution.

8. (3 marks) Determine whether the following statement is true or false.
   <u>If the statement is true, then prove it.</u>
   <u>If the statement is false, then provide a counter-example that shows that the statement is not true.</u>

   "If A and B are symmetric $n \times n$ matrices, then matrix AB is also symmetric."

9. (12 marks) A and B are 3×3 matrices and $\det(A)=\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=4$, and $\det(B)=-3$. Find

a) $\det\left(-A^T\cdot(2B)^{-3}\det(B)\right)$ b) $\det\left(5BA^{-1}-2Badj(A)\right)$

c) $\begin{vmatrix} 2a+3g & -5g & -4a+d \\ 2b+3h & -5h & -4b+e \\ 2c+3i & -5i & -4c+f \end{vmatrix}$

10. (4 marks) Evaluate the determinant $\begin{vmatrix} -2 & 1 & -5 & -1 \\ 8 & 0 & -1 & -3 \\ 1 & -1 & 6 & 2 \\ 3 & -1 & 5 & 3 \end{vmatrix}$ by row reduction. <u>**You must perform at least one row operation.**</u>

11. (9 marks) Let $\vec{u}=(2,-1,\ 3)$, $\vec{v}=(1,-2,-4)$, $\vec{w}=(3,-1,-5)$.

a) Find the orthogonal projection of the vector $\vec{w}$ on the vector $\vec{u}+\vec{v}$, that is $Proj_{\vec{u}+\vec{v}}\ \vec{w}$.

b) Find a unit vector perpendicular to $\vec{u}+\vec{v}$ and $\vec{w}$.

c) Find the area of a triangle determined by $\vec{u}+\vec{v}$ and $\vec{w}$.

12. (1+3+3 marks) Given the point $A(\ 3,-7,\ 4)$, the plane P: $x-3y+2z=4$ and the line L1: $\begin{cases} x=3+t \\ y=6-t \\ z=1-2t \end{cases}$.

a) Determine whether the line is parallel to the plane.
b) Find the point on the plane P which is closest to the point A.
c) Find the distance from the point A to the plane P.

13. (1+3 marks) Show that the planes are not parallel and find the parametric equations of the line of intersection of the planes $x-5y+3z=4$ and $2x-9y+8z=5$.

14. (3 marks) Simplify $\left[\left(\vec{a}+5\vec{b}\right)\times\left(2\vec{a}-3\vec{b}\right)\right]\cdot\left(\vec{a}-\vec{b}\right)$.

15. (6 marks) a)Show that L1: $\begin{cases} x=3+t \\ y=6-t \\ z=1-2t \end{cases}$ and L2: $\begin{cases} x=1+2u \\ y=3-u \\ z=4+3u \end{cases}$ are skew lines

b) Find the equation of the plane containing the point $A(\ 3,-7,\ 4)$ and the line L1.

16. (8 marks) Maximize $P=5x_1+4x_2+7x_3-x_4$ subject to $\begin{cases} x_1+2x_2+x_3+2x_4\le 7 \\ x_1 \qquad -x_3+3x_4\le 3 \\ \quad 3x_2+x_3 \qquad \le 4 \end{cases}$

$(x_1,x_2,x_3,x_4\ge 0)$

17. (7 marks) Minimize $C=5x_1+3x_2+18x_3$ subject to $\begin{cases} x_1+x_2+4x_3\ge 20 \\ 4x_1+3x_2\ge 10 \end{cases}$

$(x_1,x_2,x_3\ge 0)$

Answers

1. a) $x_1 = 2+3t-s,\ x_2 = -1+t+5s,\ x_3 = t,\ x_4 = s.$ b) $x_1 = 1,\ x_2 = 4,\ x_3 = 0,\ x_4 = 1.$

2. a) $A^{-1} = \begin{bmatrix} -1 & -\frac{1}{3} & \frac{5}{3} \\ 1 & 0 & -1 \\ -1 & -\frac{2}{3} & \frac{4}{3} \end{bmatrix}$; b) $x = 1,\ y = 2,\ z = 1.$

3. $y = 2$

4. $4A^{-2}$

5. a) -47; b) $X = \begin{bmatrix} 3 & 4 \\ 2 & -3 \end{bmatrix}$

6. $E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ E_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. *Other possible answers.*

7. 1) $k \neq \pm 3$; 2) $k = -3$; 3) $k = 3$

8. False.

9. a) $-\frac{1}{128}$ b) $\frac{81}{4}$; c) 40

10. -16

11. a) $\left(\frac{51}{19}, -\frac{51}{19}, -\frac{17}{19}\right)$; b) $\left(\frac{7}{\sqrt{94}}, \frac{6}{\sqrt{94}}, \frac{3}{\sqrt{94}}\right)$; c) $\sqrt{94}$.

12. a) Yes, the line is parallel to the plane; b) $(1, -1,\ 10)$; c) $2\sqrt{14}$.

13. $x = -11-13t,\ y = -3-2t,\ z = t$.

14. 0

15. b) $29x + 3y + 13z - 118 = 0$

16. $P = 43,\ x_1 = 3,\ x_2 = 0,\ x_3 = 4,\ x_4 = 0,$.

17. $C = 60,\ x_1 = 0,\ x_2 = 20,\ x_3 = 0.$