Dawson College
Department of Mathematics
Linear Algebra
201-NYC-05
Computer Science
Final Examination
Fall 2023

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Time: 9:30 to 12:30

Question	Points	Score
1	6	
2	4	
3	3	
4	9	
5	4	
6	3	
7	7	
8	5	
9	10	
10	11	
11	3	
12	9	
13	11	
14	4	
15	3	
16	8	
Total	100	

Instructions:

- Print your name and ID in the provided space.
- This exam contains 13 pages (including this cover page) and 16 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- You are only permitted to use the **Sharp EL-531**** calculator.
- This examination booklet must be returned intact.
- Good luck!

1. (a) (5 points) Use Gauss-Jordan elimination to find the general solution of the system.

$$\begin{cases} 2x_1 - 6x_2 + 4x_3 = 5 \\ x_1 - 2x_2 - x_3 + 3x_4 = 1 \\ 5x_1 - 14x_2 + 7x_3 + 3x_4 = 11 \\ 3x_1 - 8x_2 + 3x_3 + 3x_4 = 6 \end{cases}$$

(b) (1 point) Find a particular solution for which $x_1 = -6$, $x_3 = 2$.

$$\left(\begin{array}{cccc|c} 2 & -6 & 4 & 0 & 5 \\ \boxed{1} & -2 & -1 & 3 & 1 \\ 5 & -14 & 7 & 3 & 11 \\ 3 & -8 & 3 & 3 & 6 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 0 & -2 & 6 & -6 & 3 \\ 1 & -2 & -1 & 3 & 1 \\ 0 & -4 & 12 & -12 & 6 \\ 0 & -2 & 6 & -6 & 3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 0 & \boxed{-2} & 6 & -6 & 3 \\ & -4 & 12 & -12 & 6 \end{array}\right) \rightarrow \left(\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 0 & \boxed{1} & -3 & 3 & -3/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{array}{ccccc} & & x_3 = s & x_4 = t & \\ \end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & -7 & 9 & -2 \\ 0 & 1 & -3 & 3 & -3/2 \end{array}\right)$$

General Solution

$$\begin{cases} x_4 = t \\ x_3 = s \\ x_2 = -\frac{3}{2} - 3t + 3s \\ x_1 = -2 - 9t + 7s \end{cases}$$

$x_3 = s = 2$

$-6 = x_1 = -2 - 9t + 7 \cdot 2$

$\Rightarrow -6 = -2 - 9t + 14$

$\Rightarrow 9t = 12 + 6$

$\Rightarrow t = 2$

$\Rightarrow$

Particular solution

$$\begin{cases} x_4 = 2 \\ x_3 = 2 \\ x_2 = 1 \\ x_1 = -6 \end{cases}$$

$\Rightarrow x_2 = -2 - 9 \cdot 2 + 7 \cdot 3$

$= 19 - 18 = 1$

2. (4 points) Solve for X the following matrix equation:

$$\left(\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} X^T - \begin{bmatrix} 6 & 1 \\ -2 & 4 \end{bmatrix}\right)^{-1} = \begin{bmatrix} -2 & 1 \\ 7 & -3 \end{bmatrix}$$

$$(AX^T - B)^{-1} = C$$

$$\Rightarrow \quad AX^T = C^{-1} + B$$

$$\Rightarrow \quad X^T = A^{-1}C^{-1} + A^{-1}B$$

$$\Rightarrow \quad X = \left((CA)^{-1} + A^{-1}B\right)^T = \left(\begin{pmatrix} 11 & 3 \\ 15 & 4 \end{pmatrix} + \begin{pmatrix} -10 & 7 \\ -18 & 10 \end{pmatrix}\right)^T = \begin{pmatrix} 1 & -3 \\ 10 & 14 \end{pmatrix}$$

$$CA = \begin{pmatrix} -2 & 1 \\ 7 & -3 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 15 & -11 \end{pmatrix}$$

$$(CA)^{-1} = \begin{pmatrix} -11 & -3 \\ -15 & -4 \end{pmatrix}\frac{1}{44-45} = \begin{pmatrix} 11 & 3 \\ 15 & 4 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}\frac{1}{-3+4} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$$

$$A^{-1}B = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 6 & 1 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -10 & 7 \\ -18 & 10 \end{pmatrix}$$

3. (3 points) Let A and B be skew-symmetric $(n \times n)$ matrices.
Determine whether the matrix $X = A^3B^4A^3$ is symmetric, skew-symmetric or neither. Justify your answer with a proof.

$$X^T = (A^3B^4A^3)^T = (A^3)^T(B^4)^T(A^3)^T = (A^T)^3(B^T)^4(A^T)^3$$

$$= (-A)^3(-B)^4(-A)^3 = (-1)^{10}A^3B^4A^3 = A^3B^4A^3$$

So, X is symmetric

4. (6 points) Given the system of linear equations $\begin{cases} 4x + 2y + 3z = 9 \\ 2x + y + z = 3 \\ x + 3y - 2z = -11 \end{cases}$

(a) Use the adjoint matrix to find the inverse of the coefficient matrix.

(b) Use the inverse matrix to solve the system.

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & -2 \end{pmatrix}$$

$$\boxed{\operatorname{adj}(A) = \begin{pmatrix} -5 & 5 & 5 \\ 13 & -11 & -10 \\ -1 & 2 & 0 \end{pmatrix}^T} = \begin{pmatrix} -5 & 13 & -1 \\ 5 & -11 & 2 \\ 5 & -10 & 0 \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} 0 & -10 & 11 \\ 0 & -5 & 5 \\ 1 & 3 & -2 \end{vmatrix} = \begin{vmatrix} -10 & 11 \\ -5 & 5 \end{vmatrix} = -50 + 55 = 5$$

a) $$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A) = \frac{1}{5} \begin{pmatrix} -5 & 13 & -1 \\ 5 & -11 & 2 \\ 5 & -10 & 0 \end{pmatrix}$$

b) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -5 & 13 & -1 \\ 5 & -11 & 2 \\ 5 & -10 & 0 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \\ -11 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 5 \\ -10 \\ 15 \end{pmatrix} =$$

$$\boxed{\begin{aligned} A\bar{x} &= \bar{b} \\ \bar{x} &= A^{-1}\bar{b} \end{aligned}}$$

$$= \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

(c) (3 points) Use Cramer's rule to solve the system for z only.

$$|A_z| = \begin{vmatrix} 4 & 2 & 9 \\ 2 & 1 & 3 \\ 1 & 3 & -11 \end{vmatrix} = \begin{vmatrix} 0 & -10 & 53 \\ 0 & -5 & 25 \\ 1 & 3 & -11 \end{vmatrix} = \begin{vmatrix} -10 & 53 \\ -5 & 25 \end{vmatrix} = -250 + 265 = 15$$

$$\Rightarrow z = \frac{15}{5} = 3$$

5. (4 points) Find the values of a for which the system $\begin{cases} x+y+3z=3 \\ x+2y+5z=2a \\ -2x+3y+(a^2-5)z=a^2 \end{cases}$ has

(a) Exactly one solution. (b) No solutions. (c) Infinitely many solutions.

$$\left(\begin{array}{ccc|c} \boxed{1} & 1 & 3 & 3 \\ 1 & 2 & 5 & 2a \\ -2 & 3 & a^2-5 & a^2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & \boxed{1} & 2 & 2a-3 \\ 0 & 5 & a^2+1 & a^2+6 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & 1 & 2 & 2a-3 \\ 0 & 0 & a^2-9 & a^2-10a+21 \end{array}\right)$$

irrelevant

a) $a^2-9 \neq 0 \Rightarrow$
$\boxed{a \in \mathbb{R} \setminus \{3,-3\}}$

b) $a^2-9=0$ and
$a^2-10a+21 \neq 0$
$\Rightarrow \boxed{a=-3}$

c) $a^2-9=0$ and
$a^2-10a+21=0$
$\Rightarrow \boxed{a=3}$

6. (3 points) Solve for X and simplify the answer as much as possible.
$(AB)^{-1}XC^3(B^{-1})^T = (3C^{-2}AB)^{-1}(-2B^{-1}B^0C^T)^T$

$$X = AB(3C^{-2}AB)^{-1}(-2B^{-1}B^0C^T)^TB^TC^{-3}$$
$$= \underbrace{AB(AB)^{-1}}C^2\frac{1}{3}(-2)C\underbrace{(B^T)^{-1}B^T}C^{-3}$$
$$= -\frac{2}{3}C^3C^{-3} = -\frac{2}{3}I$$

7. Let $A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -3 \\ 3 & 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$.

maybe $C = \begin{pmatrix} 2 & 3 \\ -3 & -4 \end{pmatrix}$

a) (3 points) Calculate $p(AB^T)$, where $p(x) = x^2 - 3x + 2$

$$X = AB^T = \begin{pmatrix} -1 & 0 & 3 \\ 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 1 \\ -3 & -2 \end{pmatrix} = \begin{pmatrix} -10 & -9 \\ -1 & 5 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} -10 & -9 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} -10 & -9 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 109 & 135 \\ 15 & 34 \end{pmatrix}$$

$$3X = \begin{pmatrix} -30 & -27 \\ -3 & 15 \end{pmatrix}$$

$$2I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$X^2 - 3X + 2I = \begin{pmatrix} 109+30+2 & 135+27 \\ 15+3 & 34-15+2 \end{pmatrix} = \begin{pmatrix} 141 & 162 \\ 18 & 21 \end{pmatrix}$$

b) (4 points) Write C as a product of elementary matrices.

$$C^{-1} = -\frac{1}{11}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} \frac{4}{11} & \frac{3}{11} & 1 & 0 \\ \frac{1}{11} & -\frac{2}{11} & 0 & 1 \end{array}\right) R_{1\leftrightarrow 2}$$

$$\left(\begin{array}{cc|cc} \frac{1}{11} & -\frac{2}{11} & 0 & 1 \\ \frac{4}{11} & \frac{3}{11} & 1 & 0 \end{array}\right) -4R_1+R_2$$

$$\left(\begin{array}{cc|cc} 1 & -2 & 0 & 11 \\ 0 & 1 & 1 & -4 \end{array}\right) \begin{matrix} 11R_1, \\ 2R_2+R_1 \end{matrix}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & -4 \end{array}\right)$$

$$C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

8. (5 points) Evaluate the following determinant.

$$\begin{vmatrix} 2 & 3 & -7 & 5 \\ 4 & -1 & 3 & -1 \\ -1 & 1 & -3 & 2 \\ -6 & 0 & 4 & -2 \end{vmatrix} = \begin{vmatrix} 5 & 0 & 2 & -1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -3 & 2 \\ -6 & 0 & 4 & -2 \end{vmatrix} = - \begin{vmatrix} 5 & 2 & -1 \\ 3 & 0 & 1 \\ -6 & 4 & -2 \end{vmatrix} = - \begin{vmatrix} 8 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 4 & -2 \end{vmatrix}$$

$$= -8 \begin{vmatrix} 0 & 1 \\ 4 & -2 \end{vmatrix} = -8(-4) = 32$$

9. Let A and B be two 3×3 matrices such $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$, and $\det(B) = 2$. Find the following:

(a) (4 points) $\begin{vmatrix} 3f+2c & f-c & 2i+c \\ 3d+2a & d-a & 2g+a \\ 6e+4b & 2e-2b & 4h+2b \end{vmatrix}$. $= \begin{vmatrix} 5c & f-c & 2i+c \\ 5a & d-a & 2g+a \\ 10b & 2c-2b & 4h+2b \end{vmatrix}$

$-3C_2$

$$= 5 \begin{vmatrix} c & f & 2i \\ a & d & 2g \\ 2b & 2c & 4h \end{vmatrix} = 5\cdot 2\cdot 2 \begin{vmatrix} c & f & i \\ a & d & g \\ b & c & h \end{vmatrix}$$

$$= 5\cdot 2\cdot 2(-3) = -60$$

(b) (3 points) $\det\left((3BA)^{-1}(A^2)^T\det(A^2)\right)$. $= \left|\frac{1}{3}A^{-1}B^{-1}(A^2)^T(-3)^2\right|$

$$= \frac{1}{3^3}\,\frac{1}{|A|}\,\frac{1}{|B|}\,|A|^2\,9^3 = 3^3(-3)\frac{1}{2} = -\frac{81}{2}$$

(c) (3 points) $\det\left(2B^{-1}A^{-1} + \operatorname{adj}(AB)\right)$. $= \left|2(AB)^{-1} + (AB)^{-1}|AB|\right|$

$$= \left|2(AB)^{-1} + (AB)^{-1}(-3)2\right| = \left|-4(AB)^{-1}\right|$$

$$= (-4)^3\frac{1}{|AB|} = -\frac{64}{-6} = \frac{32}{3}$$

10. Consider the vectors $\vec{u} = (2, 3, -4)$, $\vec{v} = (1, -2, 1)$, $\vec{w} = (3, -1, 2)$.

(a) (4 points) Find the orthogonal projection of $\vec{u} + \vec{v}$ on $\vec{u} - \vec{w}$; that is $proj_{\vec{u}-\vec{w}}(\vec{u} + \vec{v})$.

$$\boxed{\begin{aligned}\vec{u}+\vec{v} &= (3, 1, -3)\\ \vec{u}-\vec{w} &= (-1, 4, -6)\end{aligned}}$$

$$proj_{\bar{u}-\bar{w}}(\bar{u}+\bar{v}) = \frac{(\hat{u}+\bar{v})(\bar{u}-\bar{w})}{\|\bar{u}-\bar{w}\|^2}(\bar{u}$$

$$= \frac{(3, 1, -3)\cdot(-1, 4, -6)}{1+16+36}(-1, 4, -6) = \frac{19}{53}(-1, 4, -6)$$

$$= \left(-\frac{19}{53}, \frac{76}{53}, -\frac{114}{53}\right)$$

(b) (3 points) Find the angle in degrees between vectors $\vec{u} + \vec{v}$ and $\vec{u} - \vec{w}$ Approximate your answer to two decimal places.

$$\theta = \arccos\frac{(3, 1, -3)\cdot(-1, 4, -6)}{\sqrt{9+1+9}\sqrt{1+16+36}} = \arccos\frac{19}{\sqrt{19}\sqrt{53}}$$

$$= \arccos\sqrt{\frac{19}{53}} \approx \dots$$

(c) (4 points) Find the values of k for which $\vec{u} + k\vec{v}$ is orthogonal to $\vec{u} - k\vec{v}$.

$$(\vec{u}+k\vec{v})\cdot(\bar{u}-k\bar{v}) = 0$$

$$\|\bar{u}\|^2 + k\bar{v}\cdot\bar{u} - k\bar{u}\cdot\bar{v} - k^2\|\bar{v}\|^2 = 0$$

$$k^2 = \frac{\|\bar{u}\|^2}{\|\bar{v}\|^2} \qquad k = \pm\frac{\|\bar{u}\|}{\|\bar{v}\|} = \pm\sqrt{\frac{4+9+16}{1+4+1}} = \pm\sqrt{}$$

11. (3 points) Given that $\vec{a} \cdot (\vec{b} \times \vec{c}) = -3$. Find the volume of the parallelepiped determined by th vectors $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$.

$$(a+b)\cdot((b+c)\times(c+\bar{a}))$$

$$= (\bar{a}+\bar{b})\cdot(\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \cancel{\bar{c}\times\bar{c}}^{\,0} + \bar{c}\times\bar{a})$$

$$\bar{a}\cdot(\bar{b}\times\bar{c}) + \cancel{\bar{a}\cdot(\bar{b}\times\bar{a})}^{\,0} + \cancel{\bar{a}\cdot(\bar{c}\times\bar{a})}^{\,0} +$$

$$\cancel{\bar{b}\cdot(\bar{b}\times\bar{c})}^{\,0} + \cancel{\bar{b}\cdot(\bar{b}\times\bar{a})}^{\,0} + \bar{b}\cdot(\bar{c}\times\bar{a}) \Rightarrow V = |-3-3|$$

$$= 6$$

12. Consider the points $A(2,0,1), B(3,-1,3), C(1,2,4)$.

(a) (3 points) Find the area of the triangle ABC.

(b) (3 points) Find the distance from the point C to the line AB, passing through points A and B

(c) (3 points) Find the point on the line AB closest to the point $D(4,-2,8)$.

a) $A = \frac{1}{2} \| \vec{AB} \times \vec{AC} \| = \frac{1}{2}\sqrt{49+25+1} = \frac{\sqrt{75}}{2}$

$\vec{AB} = (1,-1,2)$

$\vec{AC} = (-1,2,3)$

$\vec{AB} \times \vec{AC} = (-7,-5,1)$

b) line $P = (2,0,1) + (1,-1,2)t = (2+t, -t, 1+$

$d = \| \vec{P_0C} - \mathrm{proj}_{\vec{AB}}(\vec{P_0C}) \|$ length of projection orthogonal to

$= \| \vec{CQ} \|$ where Q is the closest point to C

$\vec{CQ} = (-1-t, 2+t, 3-2t)$

$\vec{CQ} \cdot \vec{AB} = -1-t-2-t+6-4t = 3-6t = 0$

$\Rightarrow t = \frac{1}{2} \Rightarrow \| \vec{CQ} \| = \sqrt{\frac{3^2}{2^2} + \frac{3^2}{2^2} + 2^2} = \frac{\sqrt{\ }}{2}$

c) $\vec{DQ} \cdot \vec{AB} = 2-t-2+t+7-2t = 7-2t = 0 \Rightarrow t =$

$\Rightarrow Q\left(2+\frac{7}{2}, -\frac{7}{2}, 1+7\right) \Rightarrow Q\left(\frac{11}{2}, -\frac{7}{2}, 8\right)$

13. Consider two planes $(P): x - 2y + z = 6$, $(P'): 4x - 9y - 3z = 18$ and a line $(L): \begin{cases} x = 3 - t \\ y = 1 + 2t \\ z = 1 + 5t \end{cases}$.

(a) (3 points) Find the parametric equations of the line of intersections of these planes.

$$\begin{cases} x - 2y + z = 6 \\ 4x - 0y - 3z = 18 \end{cases} \qquad \begin{cases} x - 2y + z = 6 \\ -y - 7z = -6 \end{cases}$$

$$\begin{cases} x = 6 - t + 2y = 6 - t + 12 - 14t = 18 - 15t \\ y = 6 - 7t \\ z = t \end{cases}$$

(b) (4 points) Show that the line (L) and the plane (P) are parallel, and find the distance betwe
them.

$$\vec{v} \cdot \vec{n} = (-1, 2, 5) \cdot (1, -2, 1) = -1 - 4 + 5 = 0$$

$\Rightarrow L \parallel P$

distance of $P_0(3, 1, 1)$ to plane P =

$$\frac{|3 - 2 \cdot 1 + 1 - 6|}{\sqrt{1 + 2^2 + 1}} = \frac{4}{\sqrt{6}} = \frac{2\sqrt{6}}{3}$$

(c) (4 points) Find the equation of the plane, containing the line (L) and perpendicular to the plan
(P).

$\vec{n} \perp (1, -2, 1)$ and

$\vec{n} \perp (-1, 2, 5)$ $(\vec{n} \perp L)$

$\Rightarrow \vec{n} \parallel (1, -2, 1) \times (-1, 2, 5) = (-12, -6, 0)$

$\vec{n} = (2, 1, 0)$ $P_0(3, 1, 1)$

p: $2(x - 3) + (y - 1) = 0$ or $2x + y = 7$

14. (4 points) Show that $(L_1):\begin{cases} x = 3-t \\ y = 1+2t \\ z = 1+5t \end{cases}$ and $(L_2):\begin{cases} x = 4+3s \\ y = 2+s \\ z = 3-s \end{cases}$ are skew lines and find the distance between them.

$$d = \| proj_{\vec{v}_1 \times \vec{v}_2} (\vec{P_1P_2}) \| =$$

$$\left\| \frac{(1,1,2)\cdot(1,-2,1)}{1+4+1} (1,-2,1) \right\|$$

$$= \frac{1}{6}\sqrt{1+4+1} = \frac{\sqrt{6}}{6} \neq 0$$

$$\boxed{\begin{array}{l} \vec{v}_1 = (-1,2,5) \\ v_2 = (3,1,-1) \\ \vec{v}_1 \times \vec{v}_2 = (-7,14,-7) \parallel \vec{v} \\ \vec{v} = (1,-2,1) \end{array}}$$

$$\boxed{\left.\begin{array}{l} \vec{v}_1 \times \vec{v}_2 \neq 0 \\ d \neq 0 \end{array}\right\} \Rightarrow \text{skew-lines}}$$

15. (3 points) Determine whether the following statement is true or false. If the statement is true, then prove it. If the statement is false, then provide a counterexample, that shows that the statement is not true.

"If $AB = 0$, then either $A = 0$ or $B = 0$ ".

False.

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

16. (8 points) Maximize $P = 4x + 3y - 2z$ subject to $\begin{cases} 2x + 5y + z \le 40 \\ 2x + 2y - z \le 15 \\ x + 2y - z \le 2 \end{cases}$,

where $x \ge 0, y \ge 0, z \ge 0$

$$\left[\begin{array}{ccc|c} 2 & 5 & 1 & 40 \\ 2 & 2 & -1 & 15 \\ \boxed{1} & 2 & -1 & 2 \\ \hline -4 & -3 & 2 & 0 \end{array}\right] \quad \begin{array}{c} R \\ 20 \\ 15/2 \\ 2 \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 36 \\ 0 & -2 & \boxed{1} & 11 \\ 1 & 2 & -1 & 2 \\ \hline 0 & 5 & -2 & 8 \end{array}\right] \quad \begin{array}{c} R \\ 12 \\ 11 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 7 & 0 & 3 \\ 0 & -2 & 1 & 11 \\ 1 & 0 & 0 & 13 \\ \hline 0 & 1 & 0 & 30 \end{array}\right]$$

Verifying answer

$$\begin{cases} x = 13 \\ y = 0 \\ z = 11 \end{cases}$$

$P = 4 \cdot 13 - 2 \cdot 11 = 52 - 22 = 3$