DAWSON COLLEGE

DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

LINEAR ALGEBRA 201-NYC-05 (Science)

Fall 2019

Time: 3 hours

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#	Marks
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Student Name: ____

Student ID Number: ______________________________________

This examination has 11 pages and contains 10 problems.

Each problem is worth the same amount and each part of each problem is worth the same amount.

1. Given the following matrix:

$$A=\begin{bmatrix}1&1&1&1\\2&0&1&1\\1&1&0&1\end{bmatrix}$$

a) Use Cramer's Rule to find the second unknown of the linear system whose augmented matrix is the matrix A.

b) Use Gauss-Jordan elimination to solve the homogeneous linear system whose coefficient matrix is the matrix A.

c) Evaluate $det\left(A^T A\right)$.

2. a) If A is a matrix such that both A and A^{-1} have integer entries, show that $det\left(A\right)=det\left(A^{-1}\right)$.

b) Consider the following system:

$$\begin{aligned}(5-k)x-2y-z&=1\\-2x+(2-k)y-2z&=2\\-x-2y+(5-k)z&=1\end{aligned}$$

For what values of *k*, if any, the system has: *(i)* no solution, *(ii)* a unique solution, *(iii)* infinitely many solutions.

3. Find all matrices A such that:

a) $A^T A=\begin{bmatrix}9&3&6\\3&5&4\\6&4&6\end{bmatrix}$ and A is an upper triangular matrix with positive entries on the main diagonal.

b) $A^T A=8I_2+A^2$ and A is a 2×2 skew-symmetric matrix.

c) $\left(B^{-1}(AC)\right)^{-1}=5\left(C^{-1}B\right)$ where B and C are any 3×3 invertible matrices.

4. a) Find the distance between the origin and the plane given by the equation:

$$\begin{vmatrix}x&y&z&1\\2&2&3&1\\1&2&2&1\\2&1&1&1\end{vmatrix}=0$$

Given the line $L:(x,y,z)=(7,-1,2)+t(3,-1,2)$ and the plane $P: x+2y+3z+3=0$.

b) Find the point of intersection of the line $\mathscr{L}$ with the plane $\mathscr{P}$.

c) Find the angle between the line $\mathscr{L}$ and the plane $\mathscr{P}$.

5. a) Find parameter p such that the points $A(1,-1,0), B(2,0,1), C(1,p,3) \wedge D(2,2p,5)$ *lie* in the same plane.

b) Find general and parametric equations of the plane containing the points A(3, 0, 0), B(0, 1, 0) ¿ *perpendicular* ¿ *the XY*-plane.

c) Given the points A(1, 2, -1) and B(3, 1, 0). Find the point C on the Y-axis such that the area of *the triangle ABC* is 10.

6. a) Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors in 3-space such that $u \wedge u - v$ are orthogonal. Find the norm *of* $2u+v$ if the norm of $\boldsymbol{u}$ is $\sqrt{2}$ and the norm of $\boldsymbol{v}$ is 2.

b) If A is a 3×3 matrix such that $(2A)(A^T)^2 = -I_3$ then find $det(A)$.

c) $If \begin{vmatrix} 0\,2\,b\,0 \\ 0\,0\,3 \\ a\,0\,c \end{vmatrix} = -12\, then\, find \begin{vmatrix} a & 2 \\ 3 & b \end{vmatrix}$.

7. Given the line $L:(x,y,z)=(1,0,1)+t(1,1,-1)$, the plane $\mathcal{P}$: $2x+y+2z=7$ and the point $Q(2,0,1)$.

a) Find the distance between the line $\mathcal{L}$ and the *X-axis.*

b) Find the point A on the plane $\mathcal{P}$ which is closest to the point *Q.*

c) Find parametric equations of the line which intersects both the *X-axis* and the line L at a right angle.

8. Let $W = span\{(1,1,2),(-1,0,1),(2,1,1,)\}$.

a) Show that $\{(1,1,2),(-1,0,1),(2,1,1)\}$ is linearly dependent in R^3.

b) Find a basis and the dimension of W.

c) Find a system of homogeneous linear equations whose solution space is *W.*

9. a) Show that $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 3 \end{bmatrix} \right\}$ is a basis of the vector space of all 2×2 symmetric matrices.

b) Find the coordinates of the vector $\begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix}$ relative to the basis given in part (a).

c) Find a basis of P_2 in which the vector $x+3$ has all coordinates equal to 1.

10. Determine whether the following statement is true or false. Justify your answer with a proof or a counterexample.

a) If A is a square matrix such that $A A^T$ is elementary then $A^T A$ is elementary.

b) If A is a matrix such that $A^T = -A$ then $tr(A) = 0$.

c) $W = \left\{ A \quad M_{2\times 2} \vee A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ is a subspace of $M_{2\times 2}$.

ANSWERS:

1. a) $\frac{1}{2}$ b) $\left\{\left(\frac{-1}{2}t,-\frac{1}{2}t,0,t\right)\vee t\epsilon R\right\}$ c) 0

2. a) Hint: Use the fact that the determinant of a matrix with integer entries is an integer.
   b) $(i)\,k=0\,(ii)\,k\neq 0\wedge k\neq 6\,(iii)\,k=6.$

3. a) $\begin{bmatrix}3&1&2\\0&2&1\\0&0&1\end{bmatrix}$ b) $\begin{bmatrix}0&2\\-2&0\end{bmatrix}\vee\begin{bmatrix}0&-2\\2&0\end{bmatrix}$ c) $\frac{1}{5}I_3$

4. a) $\frac{\sqrt{6}}{2}$ b) $(1,1,-2)$ c) $\frac{\pi}{6}$

5. a) $p=2$
   b) $x=3-3s,\ y=s,\ z=t$ are the parametric equations and $x+3y-3=0$ is the general equation.
   c) $C=\left(0,\frac{11\pm\sqrt{1946}}{5},0\right)$

6. a) $2\sqrt{5}$ b) $\frac{-1}{2}$ c) -8

7. a) $\frac{1}{\sqrt{2}}$ b) $\left(\frac{20}{9},\frac{1}{9},\frac{11}{9}\right)$ c) $x=\frac{3}{2},\ y=t,\ z=t$ *where* $t\epsilon R$

8. Hint: It is enough to show that $\begin{vmatrix}1&1&2\\-1&0&1\\2&1&1\end{vmatrix}=0$

9. a) Hint: It is enough to show the linear independence.
   b) $(1,2,-1)$
   c) $\{x^2,x-x^2,3\}$

10. a) False b) True c) False