DAWSON COLLEGE

DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

LINEAR ALGEBRA 201-NYC-05 (Science)

Fall 2018

Time: 3 hours

Examiners: O. Diaconescu, A. Douba, Y. Lamontagne, V. Ohanyan, S. Shahabi
B. Szczepara

Student Name: ____

Student ID Number: ___________________________________

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This examination contains 10 problems.

Each problem is worth the same amount and each part of each problem is worth the same amount.

1. Given the following matrix:

$$A=\begin{bmatrix}3124\\1020\\3102\end{bmatrix}$$

a) Solve the linear system whose coefficient matrix is the matrix A and which has a particular solution (1, 0, -1, 0).
b) Solve the linear system whose augmented matrix is the matrix A using the inverse of its coefficient matrix.

2. Consider the following system:

$$kx-y-kz=2k+1$$
$$kx-ky-2z=k+1$$
$$kx-y+kz=4k+3$$

For what values of *k*, if any, the system has: a) no solution, b) a unique solution, c) infinitely many solutions.

3. Find all matrices A such that:

a) $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}A+A\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 4\\2 & 2\end{bmatrix}$ b) $A(3I_3-2A)^{-1}=\begin{bmatrix}103\\010\\001\end{bmatrix}$

4. a) A matrix A is said to be *orthogonal* if $A^{-1}=A^T \vee, equivalently, if\ AA^T=A^TA=I$.
$If\ A\, is\, an$orthogonal matrix with integer entries show that every row of A has exactly one nonzero entry which is equal to ± 1.

b) If A and B are invertible matrices of the same size show that
$adj(AB)=(adj(B))(adj(A))$

5. a) Find all unit vectors parallel to the plane $x+2y+3z=5$ and$the\ XY$-plane.

b) Find the point P on the plane $(x,y,z)=(0,1,4)+s(0,6,-4)+t(-1,-1,1)(s,t\epsilon R)$ $which\, is\, closest\, ¿\, the\, the\, origin.$

6. a) Show that if A is an $n\times n$ skew-symmetric matrix then $B^T AB$ is also skew-symmetric $for\, any\, n\times n$ matrix B. (A matrix A is called *skew-symmetric* if$A^T=-A.¿$
b) If A and B are 3×3 matrices such that $AB^T=I\wedge det(A)=2, find\, det(A^2B)$.

7. Given the line $L:(x,y,z)=(2,2,3)+t(1,-1,-3)$, the plane $\mathscr{P}$: $3x-2y+2z=7$ and the point $A(1,1,1)$.
a) Find parametric equations of the line which contains the point A, intersects the line $\mathscr{L}$ and which is parallel to the plane $\mathscr{P}$.
b) Find parametric equations of the line which contains the point A and which intersects the line $L\, at\, the\, ¿\, angle.$

8. Let $V_n=\{A\in M_{n\times n}\vee A^T=-A\}$.
a) Show that V_n is a subspace of $M_{n\times n}$.
b) Find a basis and the dimension of V_3.

9. a) Show that $\{2x+5, x^2-3x+1, x^2+x\}$ is a basis of $P_2$and find the coordinates of *the vestor* $5x^2-x+7$relative to this basis.

b) Find all values of t such that $\{(1,-t,2),(t,3,-1),(3t,5,-4)\}$ is a linearly independent set of vectors in R^3.

10. Determine whether the following statement is true or false. Justify your answer with a proof or a counterexample.

a) The point $(1,2,1)$ is between planes $6x-3y+6z=-3 \wedge 4x-2y+4z=2$.

b) $W=\{(x,y,z)\in R^3 \vee xy=0 \wedge yz=0\}$ is a subspace of R^3.

ANSWERS:

1. a) $\{(1+2t,-8t,-1-t,t) | t\in R\}$ b) $(-2,8,1)$

2. a) $k=0 \vee k=1$ b) $k\neq 0 \wedge k\neq 1$ c) there is no such k

3. a) $A=\begin{bmatrix} 0 & \frac{7}{4} \\ 1 & \frac{1}{2} \end{bmatrix}$ b) $\begin{bmatrix} 101 \\ 010 \\ 001 \end{bmatrix}$

4. a) Hint: Use the definition of orthogonal matrix. b) Hint: Write *adjA* in terms of A^{-1}.

5. a) $\pm(\frac{2}{\sqrt{5}},-\frac{1}{\sqrt{5}},0)$ b) $(1,2,3)$

6. a) Hint: Use Properties of transpose operation. b) 2

7. a) $x=1+6t, y=1-4t, z=1-13t$ b) $x=1+\frac{17}{11}t, y=1+\frac{5}{11}t, z=1+\frac{4}{11}t$

8. a) Hint: Use the theorem. b) $\left\{\begin{bmatrix} 010 \\ -100 \\ 000 \end{bmatrix}, \begin{bmatrix} 001 \\ 000 \\ -100 \end{bmatrix}, \begin{bmatrix} 000 \\ 001 \\ 0-10 \end{bmatrix}\right\}$ is the basis and $dim(V_3)=3$

9. a) Hint: Use the theorem. The coordinates are $(1,2,3)$. b) $\{t\in R | t\neq -1 \wedge t\neq -7\}$

10. a) False b) False