DAWSON COLLEGE

Mathematics Department

Final Examination

201-SN3-RE (Integral Calculus)

Science, Science and Computer Science and Mathematics

Sections 00001-00014

Winter 2025

Date: Wednesday May 28th, 2025, Time: 14:00 – 17:00

Teachers: K. Ameur, O. Cerba, I. Gorelyshev, A. Hariton, A. Hindawi, A. Juhasz, T. Kengatharam, , A. Panait, S. Shahabi, S. Soltuz, B. Szczepara(V. Kalvin), O. Veres.

Name: ______________________________ **ID:** ______________________

- Print your name and student ID number in the space provided above;
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer, use the back of the page;
- No book, notes, graphing/programmable calculator or cellphones are permitted;
- You are only permitted to use the **Sharp EL-531**** calculator;
- Show all your work and justify all your answers;
- This examination booklet must be returned intact;

1)	9)
2)	10)
3)	11)
4)	12)
5)	13)
6)	14)
7)	15)
8)	

THIS EXAMINATION BOOKLET CONTAINS 15 PAGES (INCLUDING THIS COVER PAGE), AND 15 QUESTIONS.

(1) **(4 points)** Solve the following differential equation $\dfrac{dy}{dx} = \dfrac{y\sqrt{y}}{\sqrt{x+3}}$ given the initial condition $y(1) = 1$.

(2) **(5 points)** Using the limit of a Riemann sum, evaluate the following definite integral:

$$\int_{-2}^{1} (4x - x^2)dx$$

$$\sum_{i=1}^{n} c = cn,\ \sum_{i=1}^{n} i = \frac{n(n+1)}{2},\ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{i=1}^{n} i^3 = \frac{[n(n+1)]^2}{4}$$

(3) **(4 points)** Given the function $F(x) = \int_{x^3+2}^{\tan^2 x} \sqrt{t^3+1}dt$ find $F'(x)$. **(Do not simplify your answer)**

(4) **(3 points)** Given that $\int_3^6 4f(x)dx = 12$ and f is an odd function find $\int_{-3}^6 f(x)dx$. Assume that all the definite integrals exist.

(5) **(4+4+4 points)** Find the following integrals:

(a) **(4 points)** $\int (\sqrt{x} + 3) \ln(2x) dx$

(b) **(4 points)** $\int \cos^3(\pi x) \sin^4(\pi x) dx$

(c) **(4 points)** $\displaystyle\int \frac{4x^2}{\sqrt{64-x^2}}dx$

(6) **(5 points)** Find the average value of $f(x) = \dfrac{2025x^2}{(x+4)^3}$ on the interval $[0, 1]$.

(7) **(5+5 points)** Decide whether the following improper integrals are convergent or divergent. If convergent, find their value:

(a) **(5 points)** $\displaystyle\int_{-\infty}^{4} xe^{x/3}dx$

(b) **(5 points)** $\displaystyle\int_{2}^{4} \frac{2}{(x-1)\ln(x-1)}dx$

(8) **(6 points)** Sketch the region enclosed by **all of the following conditions** and find its area.

$$y = \frac{1}{x}, \quad y = x^3, \quad y = 0 \quad x = 2$$

(9) **(5+5 points)** Setup the integral **(do not calculate)** to find the volume of the solid obtained by rotating the region enclosed by: $y = \sqrt{x}$, $y = x^2$ using the cylindrical shells or the washer method.

(Do not calculate the integral)

(a) **(5 points)** about the x-axis.

(b) **(5 points)** about the y -axis.

(10) **(3+3 points)** Determine whether the sequence is convergent or divergent. If convergent, find its limit.

(a) **(3 points)** $(1+\frac{2}{1})^1,\ (1+\frac{2}{2})^2,\ (1+\frac{2}{3})^3,\ (1+\frac{2}{4})^4,\ ...$

(b) **(3 points)** $a_n = \dfrac{n \sin n}{n^2+1}$

(11) **(4 points)** Determine if the following series is convergent or divergent. If convergent, find its sum.

(Hint: you might need to rearrange the terms)

$$(3+\frac{1}{2})+(\frac{3}{2}+\frac{1}{6})+(\frac{3}{4}+\frac{1}{18})+(\frac{3}{8}+\frac{1}{54})+(\frac{3}{16}+\frac{1}{162})+...$$

(12) **(4+4+4+5 points)** Determine if the following series are **absolutely convergent, conditionally convergent or divergent** Justify your answers with the appropriate test.

(a) **(4 points)** $\sum_{n=1}^{\infty} ne^{-n^2}$

(b) **(4 points)** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{2n^3+3n^2+4}$

(c) **(4 points)** $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 4 \cdot 7 \cdot \ldots \cdot (3n-2)}{2^n n!}$

(d) **(5 points)** $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n} + \sqrt{n+1}}$

(13) **(5 points)** Find the radius of convergence and the interval of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{n^3+1}$$

(14) **(3+3 points)**

(a) **(3 points)** Find the Maclaurin series for $f(x) = e^{4x}$, using the definition of a Maclaurin series.

(b) **(3 points)** Evaluate $\displaystyle\int \frac{e^{4x} - 1}{x} dx$ as an infinite series.

(15) **(3 points)** Evaluate the limit:

$$\lim_{x\to 0^+} \frac{\int_0^x (e^{t^2}-1)dt}{1-\cos x}$$

Answers:

1. $y = \dfrac{4}{(2\sqrt{x+3} - 6)^2}$

2. -9

3. $-3x^2\sqrt{(x^3+2)^3+1} + 2\tan x \sec^2 x \sqrt{\tan^6 x+1}$

4. 3

5. (a) $\left(\frac{2}{3}x^{3/2} + 3x\right)\ln(2x) - \frac{4}{9}x^{\frac{3}{2}} - 3x + C$

(b) $\frac{1}{\pi}\left(\dfrac{\sin^5(\pi x)}{5} - \dfrac{\sin^7(\pi x)}{7}\right) + C$

(c) $128 \arcsin\left(\frac{x}{8}\right) - 2x\sqrt{64-x^2} + C$

6. 6.37

7. (a) convergent: $3e^{4/3}$

(b) divergent

8. $\frac{1}{4} + \ln 2$

9. (a) $V = \int_0^1 [\pi(\sqrt{x})^2 - \pi(x^2)^2]\,dx$

(b) $V = \int_0^1 2\pi x(\sqrt{x} - x^2)\,dx$

10. (a) convergent: e^2

(b) convergent: 0

11. ~~(a)~~ convergent: sum $= \frac{27}{4}$

12. (a) abs conv

(b) abs conv

(c) div

(d) conditionally conv

13. $R = 1$, $[2,4]$

14. (a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n x^n}{n!}$

(b) $4x + 4x^2 + \frac{32}{9}x^3 + \frac{8}{3}x^4 + \ldots + C$

15. 0