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DAWSON COLLEGE
MATHEMATICS DEPARTMENT
Linear Algebra(SCIENCE)

I confirm that I have read and understood the College's Academic Integrity Document and will adhere to the principles of academic integrity while writing this exam.

201-SN4-RE S01-S02 Name: ______________________________

Fall 2025

Final Examination ID#: ______________________________

9:30-12:30 on December 15$^{\text{th}}$, 2025

Time Limit: 3 hours Instructor: K. Ameur,

- This exam contains 15 pages (including this cover page) and 16 problems. Check to see if any pages are missing.
- Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, and please indicate that you have done so.
- Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your solutions in a legible and orderly manner.
- This examination booklet must be returned intact.
- You are only permitted to use the Sharp EL-531** Calculator
- Good luck!

Question	Points	Score
1	8	
2	8	
3	5	
4	9	
5	4	
6	4	
7	4	
8	7	
9	3	
10	3	
11	8	
12	9	
13	9	
14	4	
15	9	
16	6	
Total:	100	

1. Consider the system $\begin{cases} x_1 + 2x_2 + 4x_3 + x_4 - x_5 = 1 \\ 2x_1 + 4x_2 + 8x_3 - x_4 + 4x_5 = 5 \\ x_1 + 3x_2 + 7x_3 + 3x_5 = -2 \end{cases}$

   (a) (6 marks) Solve the system using Gauss Jordan elimination.

   (b) (2 marks) Give a particular solution such that $x_1 = 7$, and $x_4 = -5$.

2. (8 marks) Find the values of k for which the following system has:

$$\begin{cases} x-y+z=-1 \\ -x+k^2y+kz=-k \\ 2x-2y+(k^2-k)z=k-1 \end{cases}$$

(a) Exactly one solution.
(b) No solutions.
(c) Solutions with one free variable.
(d) Solutions with two free variables

3. (5 marks) Solve for the matrix A the following matrix equation:

$$\left(\begin{bmatrix}1 & 4\\1 & 2\end{bmatrix} - (2A^{-1})^T\right)^{-1}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} = \frac{1}{2}A^T$$

4. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 5 & -4 \\ 1 & -1 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

   (a) (6 marks) Solve the system $AX = b$, by inverting the coefficient matrix.

   (b) (3 marks) Use Use Cramer's rule to solve the system $AX = b$ for y only.

5. (4 marks) Write $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ as a product of elementary matrices.

6. (4 marks) Let A and B be two matrices of size $(2n+1) \times (2n+1)$, such that $AB + A^T B^T = \mathbf{0}$. Show that the homogeneous $ABX = \mathbf{0}$ has nontrivial solutions.

7. (4 marks) Evaluate the following determinant. (Hint: first use elementary operations)

$$\begin{vmatrix} 6 & 3 & 0 & -3 \\ -3 & -2 & 2 & 2 \\ -7 & 2 & 6 & 3 \\ -5 & -4 & 2 & 4 \end{vmatrix}$$

8. Let A and B be two 3×3 matrices such that $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$, and $\det(B) = 2$. Find the following:

(a) (4 marks) $\begin{vmatrix} f-c & 3f+2c & 2i+c \\ d-a & 3d+2a & 2g+a \\ 2e-2b & 6e+4b & 4h+2b \end{vmatrix}$.

(b) (3 marks) $\det\left(2B^{-1}A + \operatorname{adj}(A^{-1}B)\right)$.

9. (3 marks) If the volume of the parallelpiped determined by $\mathbf{u}$ and $\mathbf{v}$ and $\mathbf{w}$ is equal to 9. Find all the possible values of the scalar triple product of the vectors $2\mathbf{u}+3\mathbf{v}$, $3\mathbf{v}-\mathbf{w}$, and $\mathbf{u}-\mathbf{v}$.

10. (3 marks) Let $\mathbf{u}$ and $\mathbf{v}$ be unit vectors, such that $\mathbf{u}$ is orthogonal to $3\mathbf{v}+\mathbf{u}$. Find $||4\mathbf{u}-3\mathbf{v}||$.

11. Consider the points $A(3,1,2)$, $B(0,-1,4)$, and $C(1,-1,3)$.

    (a) (3 marks) Find the angle in radian at the vertex C of the triangle ABC.

    (b) (3 marks) Find the area of the triangle ABC.

    (c) (2 marks) Find the orthogonal projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$.

12. Consider the lines $(L): \begin{cases} x = 1+2t \\ y = 3-t \\ z = 5+t \end{cases}$, $(L'): \begin{cases} x = 1-4t \\ y = 1+2t \\ z = 3-2t \end{cases}$, $t \in \mathbb{R}$

    (a) (3 marks) Find the distance between the lines (L) and (L').

    (b) (3 marks) Find the point on the plane $(P): 2x-3y-2z-14=0$ closest to the point $A(-3,2,4)$.

    (c) (3 marks) Find the general equation of the plane that contains (L) and is perpendicular to the plane (P)

13. (a) (5 marks) Show that the lines $(L_1):\begin{cases} x=-2+t \\ y=1+t \\ z=4 \end{cases}$, and $(L_2):\begin{cases} x=5+s \\ y=1 \\ z=9+2s \end{cases}$, $t,s\in\mathbb{R}$ are skew lines, and find the distance between them.

(b) (4 marks) Find the parametric equations of the line that intersects (L_1) and (L_2) at right angles.

14. (4 marks) Let $\mathbf{v_1} = (3k+2, 1, 1)$, $\mathbf{v_2} = (1, 3k+2, 1)$ and $\mathbf{v_3} = (1, 1, 3k+2)$. Find the values of k for which the vectors $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are linearly independent in $\mathbf{R}^3$

15. Let $\mathbf{v_1} = (2,1,-1)$, $\mathbf{v_2} = (1,-1,1)$ and $\mathbf{v_3} = (4,1,1)$
    (a) (3 marks) Determine if $\mathbf{z} = (3,2,0)$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$
    (b) (3 marks) Show that $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ form a basis for $\mathbb{R}^3$
    (c) (3 marks) Find the coordinates of $\mathbf{W} = (-1,2,-4)$ relative to the basis B

16. Determine whether each statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof.

    (a) (3 marks) If A and B are two matrices that commute, then $(AB)^2 = A^2B^2$

    (b) (3 marks) If A, and B are two matrices such that the product AB is defined and det(AB) $\neq 0$, then A and B are invertible.