DAWSON COLLEGE

Mathematic Department

Final Examination

Remedial Activities For Secondary IV Mathematics
(201-016-50)
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May 17, 2016

Name: ______________________________________

ID Number: ______________________________________

Instructions:

- ▷ There are a total of **17 questions** for **100 marks** (tentative mark scheme) on this examination.
- ▷ Show all your work clearly and justify briefly your answers.
  Partial, incomplete, or unjustified answers will NOT receive full marks.
  In particular trial and errors/guessing is NOT an acceptable method.
- ▷ Write exact final answers (using fractions when necessary, NO DECIMALS!).
- ▷ Solve the problems in the space provided for each question.
  If more space is needed, write *PTO*, and continue on the back of the page. A supplementary page is included at the end.
- ▷ You are only permitted to use the **Sharp EL-531XG** calculator.
- ▷ This examination booklet must be returned intact. (**13 pages**).

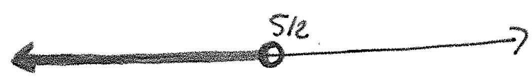
[*12 pts*] 1. Solve for x:

(a) $-(4x-2)+2(-3x+5)=3$

$$-4x+2-6x+10=3$$
$$-10x+12=3$$
$$-12 \quad -12$$
$$\frac{-10x}{-10}=\frac{-9}{-10}$$
$$\boxed{x=\frac{9}{10}}$$

(b) $3(x+5)>5(x+2)$

$$3x+15>5x+10$$
$$-3x \qquad -3x$$
$$15>2x+10$$
$$-10 \qquad -10$$
$$\frac{5}{2}>\frac{2x}{2} \qquad x<\frac{5}{2} \quad \text{or} \quad \boxed{x\in\left(-\infty,\frac{5}{2}\right)}$$



(c) $4x^3-8x^2-20x=0$

$$4x(x^2-2x-5)=0 \qquad \Delta=(-2)^2-4(1)(-5)=24=2^2\cdot 6$$

or

$$\boxed{x=0} \qquad x^2-2x-5=0$$
$$x=\frac{+2\pm\sqrt{2^2\cdot 6}}{2(1)}=\frac{2\pm2\sqrt{6}}{2}=\frac{\cancel{2}(1\pm\sqrt{6})}{\cancel{2}}$$
$$\boxed{x=1\pm\sqrt{6}}$$

[3 pts] 2. Solve for E in the formula $r = (1+E)a$.

$$\frac{r}{a} = \frac{(1+E)\cancel{a}}{\cancel{a}}$$

$$\frac{r}{a} - 1 = 1 + E - 1$$

$$\boxed{\frac{r}{a} - 1 = E}$$

alt

$$r = (1+E)a$$

$$r - a = a + Ea - a$$

$$\frac{r-a}{a} = \frac{Ea}{a}$$

$$\boxed{\frac{r-a}{a} = E}$$

[8 pts] 3. Factor completely:

(a) $4x^2 + 5x - 6 = 4x^2 + 8x - 3x - 6 = 4x(x+2) - 3(x+2)$

$= \boxed{(4x-3)(x+2)}$

$m + n = 5 = 8 - 3$

$mn = -24 = 8(-3)$

(b) $x^3y - xy^3 = xy(x^2 - y^2) = \boxed{xy(x-y)(x+y)}$

[4 pts] 4. Divide and simplify:

$$\frac{2x^2-6x}{x^2-1} \div \frac{x^2-2x-3}{(x-1)^2} = \frac{2x^2-6x}{x^2-1} \cdot \frac{(x-1)^2}{x^2-2x-3}$$

$2x^2-6x = 2x(x-3)$

$x^2-1 = (x-1)(x+1)$

$x^2-2x-3 = (x-3)(x+1)$

$m+n = -2 = -3+1$

$mn = -3 = (-3)(1)$

$$= \frac{2x\cancel{(x-3)}\cancel{(x-1)}(x-1)}{\cancel{(x-1)}(x+1)\cancel{(x-3)}(x+1)}$$

$$= \boxed{\frac{2x(x-1)}{(x+1)^2}}$$

[4 pts] 5. Subtract the rational expression and simplify:

$$\frac{(x-4)\,3x}{(x-4)(x^2+x-12)} - \frac{x\,(x-3)}{(x^2-16)(x-3)} = \frac{3x(x-4)-x(x-3)}{(x-4)(x+4)(x-3)}$$

$x^2+x-12 = (x+4)(x-3)$

$m+n = 1 = 4-3$

$mn = -12 = 4(-3)$

$x^2-16 = (x+4)(x-4)$

$$= \frac{3x^2-12x-x^2+3x}{(x-4)(x+4)(x-3)}$$

$$= \frac{2x^2-9x}{(x-4)(x+4)(x-3)}$$

$$= \boxed{\frac{x(2x-9)}{(x-4)(x+4)(x-3)}}$$

[4 pts] 6. Solve the linear system $\begin{array}{l} 2\times \\ 3\times \end{array}\begin{cases} 2x - 3y = -5 & (1) \\ 3x + 2y = 12 & (2) \end{cases}$

$$\begin{array}{rl} & 4x - 6y = -10 \\ + & 9x + 6y = 36 \\ \hline & 13x = 26 \end{array}$$

$$\frac{13x}{13} = \frac{26}{13}$$

$$\boxed{x = 2}$$

Take $x = 2$ in (2) $\Rightarrow$ $6 + 2y = 12$

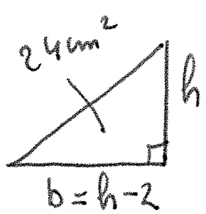
$-6 \qquad -6$

$$\frac{2y}{2} = \frac{6}{2}$$

$$\boxed{y = 3}$$

Solution : $\boxed{(2, 3)}$

[4 pts] 7. The base of a right triangle is 2 cm shorter than its height. The area of the triangle is 24 cm^2. Find the lengths of the base and the height. $\left(A = \dfrac{bh}{2}\right)$.



$$2\times \quad 24 = \frac{h(h-2)}{2} \quad \times 2$$

$$48 = h(h-2)$$

$$48 = h^2 - 2h$$

$-48 \qquad -48$

$$0 = h^2 - 2h - 48$$

$m + n = -2 = -8(6)$
$mn = -48 = -8(6)$

$$0 = (h-8)(h+6)$$

$h - 8 = 0 \qquad h + 6 = 0$

$h = 8 \qquad h = -6 \leftarrow$ discard (length are positive).

$\Rightarrow b = 8 - 2 = 6$

The length of the base is 6 cm and that of the height is 8 cm.

[8 pts] 8. Solve for x:

(a) $\frac{x-2}{x-5} = \frac{15}{x^2-5x}$ $\Rightarrow$ $\left(\frac{x-2}{x-5} = \frac{15}{x(x-5)}\right) \cdot x(x-5)$

$$\frac{(x-2)x\cancel{(x-5)}}{\cancel{x-5}} = \frac{15\cancel{x}\cancel{(x-5)}}{\cancel{x}\cancel{(x-5)}}$$

$$x(x-2) = 15$$

$$x^2 - 2x = 15$$
$-15 \quad -15$

$$x^2 - 2x - 15 = 0 \qquad m+n = -2 = -5+3$$
$$mn = -15 = (-5)3$$

$$(x-5)(x+3) = 0$$

$x-5=0$ $\quad$ $x+3=0$

$x=5$ $\quad$ $x=-3$

Check: $\underline{x=5}$: $x-5 = 5-5 = 0 \Rightarrow$ extraneous

$\underline{x=-3}$: $x-5 = -3-5 = -8 \neq 0$

$x^2-5x = (-3)^2 - 5(-3) = 9+15 = 24 \neq 0$ ✓

Solution: $\boxed{x=-3}$

(b) $\sqrt{5x+1} - x = 1$
$+x \quad +x$

$$\left(\sqrt{5x+1}\right)^2 = (1+x)^2$$

$$5x+1 = 1+2x+x^2$$
$-5x \quad -1 \quad -1 \quad -5x$

$$0 = -3x + x^2$$

$$0 = x(-3+x)$$

$x=0$ $\quad$ $-3+x=0$

$x=3$

Check: $x=0$: $\sqrt{5(0)+1} - 0 = \sqrt{1} - 0 = 1$ ✓

$x=3$: $\sqrt{5(3)+1} - 3 = \sqrt{16} - 3 = 4-3 = 1$ ✓

Solutions: $\boxed{x=0,\ x=3}$

[4 pts] 9. Simplify, expressing the result with positive exponents only.

$$\left(\frac{2^3a^{-2}\sqrt{b}}{2^{-2}a^4b^{-5/2}}\right)^{-2} = \left(\frac{2^3\, a^{-2}\, b^{1/2}}{2^{-2}\, a^4\, b^{-5/2}}\right)^{-2}$$

$$= \frac{2^{-6}\, a^4\, b^{-1}}{2^4\, a^{-8}\, b^5}$$

$$= \frac{a^{4+8}}{2^{4+6}\, b^{5+1}}$$

$$= \boxed{\frac{a^{12}}{1024\, b^6}}$$

[4 pts] 10. Rationalize the denominator and simplify

$$\frac{(2-\sqrt{10})(5\sqrt{2}+2\sqrt{5})}{(2-\sqrt{10})(2+\sqrt{10})} = \frac{2(5\sqrt{2})+2(2\sqrt{5})-(\sqrt{10})5\sqrt{2}-(\sqrt{10})2\sqrt{5}}{2^2-(\sqrt{10})^2}$$

$$= \frac{10\sqrt{2}+4\sqrt{5}-5\sqrt{20}-2\sqrt{50}}{4-10}$$

$$= \frac{10\sqrt{2}+4\sqrt{5}-5\sqrt{2^2\cdot 5}-2\sqrt{5^2\cdot 2}}{-6}$$

$$= \frac{10\sqrt{2}+4\sqrt{5}-10\sqrt{5}-10\sqrt{2}}{-6}$$

$$= \frac{-6\sqrt{5}}{-6}$$

$$= \boxed{\sqrt{5}}$$

11. The cost, y, of producing x units of a product is given by the linear function $y = ax + b$. It costs \$9,280 to produce 200 units and \$5,905 to produce 125 units.

[4 pts] (a) Find the linear function.

$$
\begin{array}{rl}
 & 9280 = a(200) + b \quad (1) \\
- & 5905 = a(125) + b \quad (2) \\
\hline
 & 3375 = 75a \\
 & \boxed{a = 45}
\end{array}
$$

Take $a = 45$ in (1):

$9280 = 45(200) + b$

$9280 = 9000 + b$

$-9000 \qquad -9000$

$\boxed{280 = b}$

Linear function: $\boxed{y = 45x + 280}$

[2 pts] (b) How much does it cost to produce 100 units?

$x = 100$

$y = 45(100) + 280 = 4500 + 280$

$y = 4780$

It costs \$4,780 to produce 100 units.

[2 pts] (c) What is the cost per unit?

The cost per unit is \$45.

[8 pts] 12. Solve for x:

(a) $16^{x-1} = 8^{2x+1}$

$(2^4)^{x-1} = (2^3)^{2x+1}$

$2^{4x-4} = 2^{6x+3}$

$4x - 4 = 6x + 3$

$-4x \qquad -4x$

$-4 = 2x + 3$

$-3 \qquad -3$

$\frac{-7}{2} = \frac{2x}{2} \Rightarrow \boxed{x = -\frac{7}{2}}$

(b) $\log_8 x = -2 \iff 8^{-2} = x$

$\frac{1}{8^2} = x$

$\boxed{x = \frac{1}{64}}$

[4 pts] 13. Find an equation for the line perpendicular to the line $2x + 3y = 8$ and through the origin.

Slope of $2x+3y=8$: $\quad 2x + 3y = 8 \Rightarrow \frac{3y}{3} = \frac{-2x+8}{3}$

$-2x \qquad -2x$

$\Rightarrow y = \boxed{-\frac{2}{3}} x + 8$ ← slope.

Slope of the required line, m: $\quad m\left(-\frac{2}{3}\right) = -1$ (perpendicular)

$\boxed{m = \frac{3}{2}}$

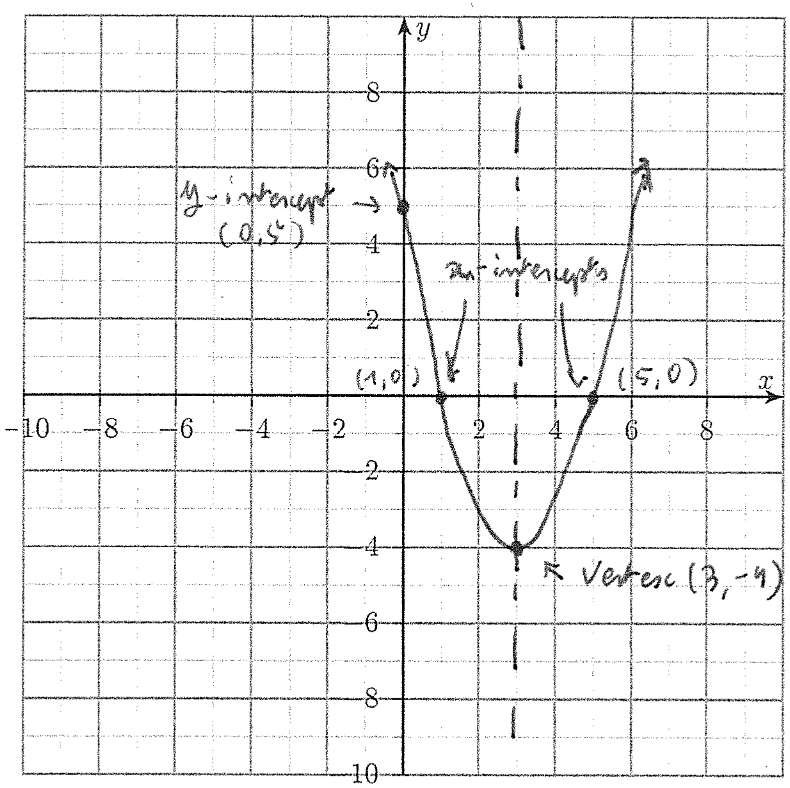
Point-slope equation: $\quad y - 0 = \frac{3}{2}(x - 0)$ (origin $= (0,0)$)

$\Rightarrow \boxed{y = \frac{3}{2}x}$

[*5 pts*] 14. Find the y-intercept, x-intercepts, and the vertex, and sketch the graph of the parabola

$$y = x^2 - 6x + 5$$

Label clearly your drawing.



y-intercept: $(0,5)$

x-intercepts: $(5,0), (1,0)$

Axis of symmetry: $x = 3$

Vertex: $(3,-4)$

y-intercept: Take $x = 0$, $y = 0^2 - 6(0) + 5 = 5$

x-intercept(s): take $y = 0$, $0 = x^2 - 6x + 5$

$$0 = (x-1)(x-5)$$

$$x - 1 = 0 \qquad x - 5 = 0$$

$$x = 1 \qquad x = 5$$

axis of symmetry: $x = \frac{-(-6)}{2(1)}$

$$x = \frac{6}{2}$$

$$x = 3$$

Vertex: $x = 3$, $y = 3^2 - 6(3) + 5 = 9 - 18 + 5 = -4$

[12 pts] 15. Consider the functions $f(x) = \dfrac{4}{x+3}$ and $g(x) = 2x^2 - 3$

(a) State the domains of $f(x)$ and $g(x)$.

Domain of f : $x + 3 \neq 0 \Rightarrow x \neq -3$

$\boxed{(-\infty, -3) \cup (-3, \infty)}$ (or $\mathbb{R} \setminus \{-3\}$)

Domain of g : $\mathbb{R}$ (quadratic function)

(b) Evaluate $3f(3) - 2g(-2) = 3 \cdot \dfrac{4}{3+3} - 2\left(2(-2)^2 - 3\right)$

$$= \frac{12}{6} - 2(8-3)$$

$$= 2 - 2(5)$$

$$= 2 - 10$$

$$= \boxed{-8}$$

(c) Evaluate and simplify $g(1 + h)$.

$$g(1+h) = 2(1+h)^2 - 3 = 2(1+2h+h^2) - 3$$

$$= 2 + 4h + 2h^2 - 3$$

$$= \boxed{2h^2 + 4h - 1}$$

[4 pts] 16. Find two consecutive integers such that the sum of the smallest one and the square of the biggest one is 55. Give all possible solution(s).

Let x and $x+1$ be the two consecutive integers.

$$x+(x+1)^2=55$$

$$x+x^2+2x+1=55$$

$$x^2+3x+1=55$$
$$-55 \quad -55$$

$$x^2+3x-54=0 \qquad m+n=3=9-6$$
$$mn=-54=9(-6)$$

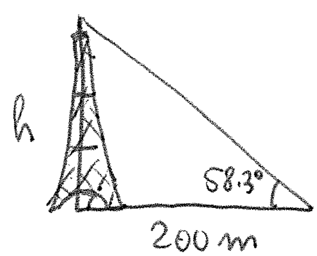
$$(x+9)(x-6)=0$$

$$x+9=0 \qquad x-6=0$$

$$\boxed{\begin{array}{l} x=-9 \\ x+1=-8 \end{array}} \qquad \boxed{\begin{array}{l} x=6 \\ x+1=7 \end{array}}$$

The two integers are $\underline{-9 \text{ and } -8}$ or $\underline{6 \text{ and } 7}$

[4 pts] 17. The angle of elevation to the top of the Eiffel Tower from a point on the ground 200m from the away is 58.3°. Find the height of the Eiffel Tower (round your answer to the nearest integer).



$$\frac{h}{200}=\tan 58.3$$

$$\Rightarrow h=200\tan 58.3 \simeq 324\,m$$

The height of the Eiffel Tower is 324 m

Supplementary / draft page