

DAWSON COLLEGE
MATHEMATICS DEPARTMENT

Final Examination
Fall 2016

Calculus 2 (201-203-DW)

Date: Monday, December 19th, 2016 at 9:30am

Instructors: C. Farnesi, A. Juhasz, I. Rajput

1. **[5 marks]** Burger sales (in hundreds of dollars) at Luke's Diner model the function:

$$S(x) = \frac{6x^5 + 3x^2 + 2}{x^3}$$

where x is the number of days since the opening of the diner. Find the **average burger sales** over the interval of day 1 to day 3 since the opening.

2. **[7 marks]** Use the limit definition of the definite integral (Riemann Sums) to evaluate

$$\int_1^6 (x^2 - 3x) dx$$

No marks will be given for using the rules of anti-differentiation.

3. **[6 marks]** Find the area of the region completely enclosed by the graphs of $f(x) = 2x$ and $g(x) = x^2 - x + 2$.
4. **[6 marks]** Each month, the quantity demanded x (in tens of units) of allergen-friendly cupcakes is related to the unit price p (in dollars) by the demand function $D(x) = -0.5x + 20$, and the supply function $S(x) = 0.1x + 5$.
- a) Find the unit market price at equilibrium.
b) Find the consumers' surplus if the unit market price is set at equilibrium.
5. **[5 marks]** Use Simpson's Rule with $n = 4$ to approximate to 3 decimal places the value of the definite integral

$$\int_0^8 \frac{20}{x^3 + 1} dx$$

6. [20 marks] Solve the following integrals:

a. $\int \left(\sec^2 3x - \frac{5}{\sqrt{1-x^2}} \right) dx$

b. $\int \frac{3x^5}{(x^3+5)^3} dx$

c. $\int (x+5) \cos 5x dx$

d. $\int \frac{x^2-2x+5}{(x-2)(x^2+1)} dx$

7. [6 marks] Evaluate the limit, if it exists:

$$\lim_{x \rightarrow 1^+} \left(\frac{3}{\ln x} - \frac{3}{x-1} \right)$$

8. [6 marks] Evaluate the integral if it converges, or show that it diverges:

$$\int_2^{\infty} \frac{4}{\sqrt{x+7}} dx$$

9. [6 marks] Given the initial condition $y(1) = -2$, use separation of variables to find the particular solution of the differential equation:

$$xy'(2y^2 + 5y) = 3y \ln x$$

10. [6 marks] Find the third Taylor Polynomial of the function $f(x) = e^{-2x+1}$ at $x = \frac{1}{2}$.

11. [5 marks] Determine if the **sequence** $\{a_n\} = \left\{ \frac{2-e^{3n}}{4e^{3n}+5} \right\}$ converges or diverges.

12. [7 marks] Show that the following series converges, and then find its sum:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n-2}}$$

13. [15 marks] Determine if each of the following series is convergent or divergent. State the test used.

a) $\sum_{n=1}^{\infty} \frac{21n^8+8n^2-5}{-12+6n^3+3n^8}$

$$b) \sum_{n=2}^{\infty} \frac{5}{\sqrt[3]{n^2-2}}$$

$$c) \sum_{n=1}^{\infty} n e^{-n^2}$$

Answers

1. Average sales are 2,809.24\$

2. $\frac{115}{6}$

3. Area is 0.17 *units*²

4. a) $\bar{p} = 7.50\$$

b) 1,562.50\$

5. 19.941

6. a) $\frac{1}{3} \tan 3x - 5 \arcsin x + C$

b) $\frac{-1}{x^3+5} + \frac{5}{2(x^3+5)^2} + C$

c) $\frac{1}{5}(x+5) \sin 5x + \frac{1}{25} \cos 5x + C$

d) $\ln|x-2| - 2 \arctan x + C$

7. $\frac{3}{2}$

8. The integral diverges.

9. $y^2 + 5y = \frac{3}{2}(\ln x)^2 - 6$

10. $P_3(x) = 2 - 2x + 2\left(x - \frac{1}{2}\right)^2 - \frac{4}{3}\left(x - \frac{1}{2}\right)^3$

11. The sequence converges. (It has a limit of $\frac{-1}{4}$).

12. $S = \frac{125}{6}$

13. a) The series diverges by the divergence test. (The limit is $7 \neq 0$).

b) The series diverges by the comparison test (to a p-series with $p = \frac{2}{3}$).

c) The series converges by the integral test.