Dawson College

Mathematics Department

Final Examination

201-MA1-DW Calculus-I Social Science

Fall - 2023

Student Name: ________________________________

Student I.D. #: ________________________________

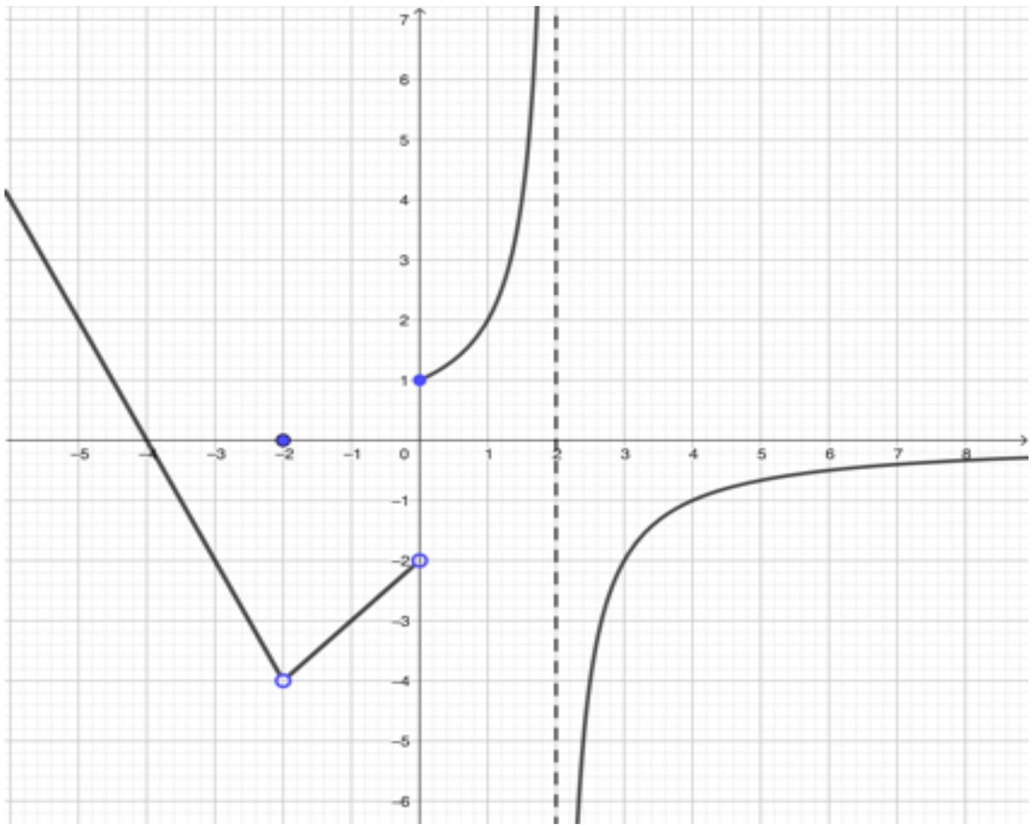
Instructor Name: ________________________________

Instructors: G. Chu, I. Gorelyshev , A. Hindawi, M. Moodi, N. Sabetghadam, I. Rajput, O. Veres

INSTRUCTIONS :

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work clearly and justify your answers.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- You are only permitted to use the Sharp EL-531** calculator.
- Verify that your final examination copy has a total of 14 questions on 13 pages, including this cover page.
- Please <u>ensure</u> that you have a complete exam package before starting.
- The examination must be returned intact.

1. [a, b, c, d, e, f, g: 3.5 marks + h: 1.5 marks] The graph of $y = f(x)$ is given below. Answer the following questions. Use DNE if the limit does not exist and $-\infty$ or ∞ if needed.



a. $\lim_{x \to -2} f(x) =$

b. $\lim_{x \to 0} f(x) =$

c. $\lim_{x \to 2} f(x) =$

d. $f(-2) =$

e. $\lim_{x \to \infty} f(x) =$

f. Find the equation of the vertical asymptote, if any.

g. Find the equation of the horizontal asymptote, if any.

h. Find the x value(s) where the function is discontinuous.

2. [10 marks] Use algebraic techniques to evaluate the following limits if they exist, show your work, and write $-\infty$ or ∞ if needed.

a. $\lim\limits_{x \to 2} \frac{x-2}{\sqrt{x+2}-2}$

b. $\lim\limits_{x \to 3} \frac{x-\frac{9}{x}}{x^2-x-6}$

3. [4 marks] The cost (in dollars) of removing $p\%$ of the air pollutants in the stack emission of a utility company that burns coal is modeled by:

$$C(p) = \frac{120{,}000p}{100 - p} \qquad 0 \leq p < 100$$

a. Find the costs of removing 15% and 85%.

b. Find

$$\lim_{p \to 100^-} \left(\frac{120{,}000p}{100 - p} \right)$$

4. [5 marks] The concentration C (in mg/cm^3) of a certain drug in a patient's bloodstream is:

$$C(t) = \frac{0.15t}{t^2 + 1}$$

Where t is the time (in h) after the drug is administered.

Find $\lim_{t \to \infty} C(t)$ and interpret your result.

5. [6 marks] Find the value(s) of discontinuity of the function. Justify your answer.

$$f(x) = \begin{cases} \dfrac{5}{(x+7)(x-2)} & if \quad x < -3 \\ \dfrac{x+7}{7} & if \quad x \geq -3 \end{cases}$$

6. [6 marks] Use the limit definition of the derivative of a function to find $f'(x)$ for $f(x) = 3 - 5x^2$.

7. [16 marks] Find the derivative of each function. DO NOT simplify your answer.

a. $y = \sqrt[4]{x^3} - 3x^{\pi} + \frac{4}{e^x} + e^{\pi}$

b. $f(x) = \sqrt{\frac{5x+2}{x^4+3}}$

c. $g(x) = \tan^2(3x) + \sin x\ \cos(5x)$

d. $y = (x^4 + 3)^{4x-1}$ Use logarithmic differentiation.

8. [5 marks] Given $f(x) = ln(7 - 5x)$.

   a. find. $f''(x)$

   b. Find $f''(0)$

9. [6 marks] A manufacturing company estimates that the revenue in thousands of dollars form the sale of x units of their product is given by:

$$R(x) = 75xe^{(1-0.02x)}$$

Find the marginal revenue when $x = 25$.

10. [6 marks] Suppose the unit price of a product (in dollars) is related to the weekly demand x (in units of a thousand) by the equation $p = 324 - x^2$.
How fast is the weekly demand changing if the price is increasing at the rate of $4/item/week and $p = 68$, $x = 16$? Round the answer to the nearest three decimal places.

11. [8 marks] For a certain product the demand function is:

$$p = 4000 - 0.5x$$

and the cost function is:

$$C(x) = 12000 + 50x \qquad 0 \le x \le 6000$$

where x is the number of units that can be sold at a price of p dollars per unit and $C(x)$ is the total cost (in dollars) of producing x units.

a. [1 mark] Find the revenue function in terms of x.

b. [1 mark] Find the profit function in terms of x.

c. [3 marks] Find the marginal profit at $x = 1200$. Interpret your result.

d. [3 marks] Find the level of production that will yield a maximum profit. What is the maximum profit?

12. [6 marks] Given the function $y = f(x)$ defined by the equation: $2xy^2 - x^3 = x^2y - 20$.

a. Find y'.

b. Find an equation of the tangent line to graph of the function $y = f(x)$ at $(-1,3)$.

13. [6 marks] A rectangular storage container with an open top is to have a volume of $30m^3$. The length of its base is three times the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

14. [11 marks] Consider the function $f(x) = 2x^5 - 10x^4$.

a. [2 mark] Find the x-intercept(s) and y-intercept of f.

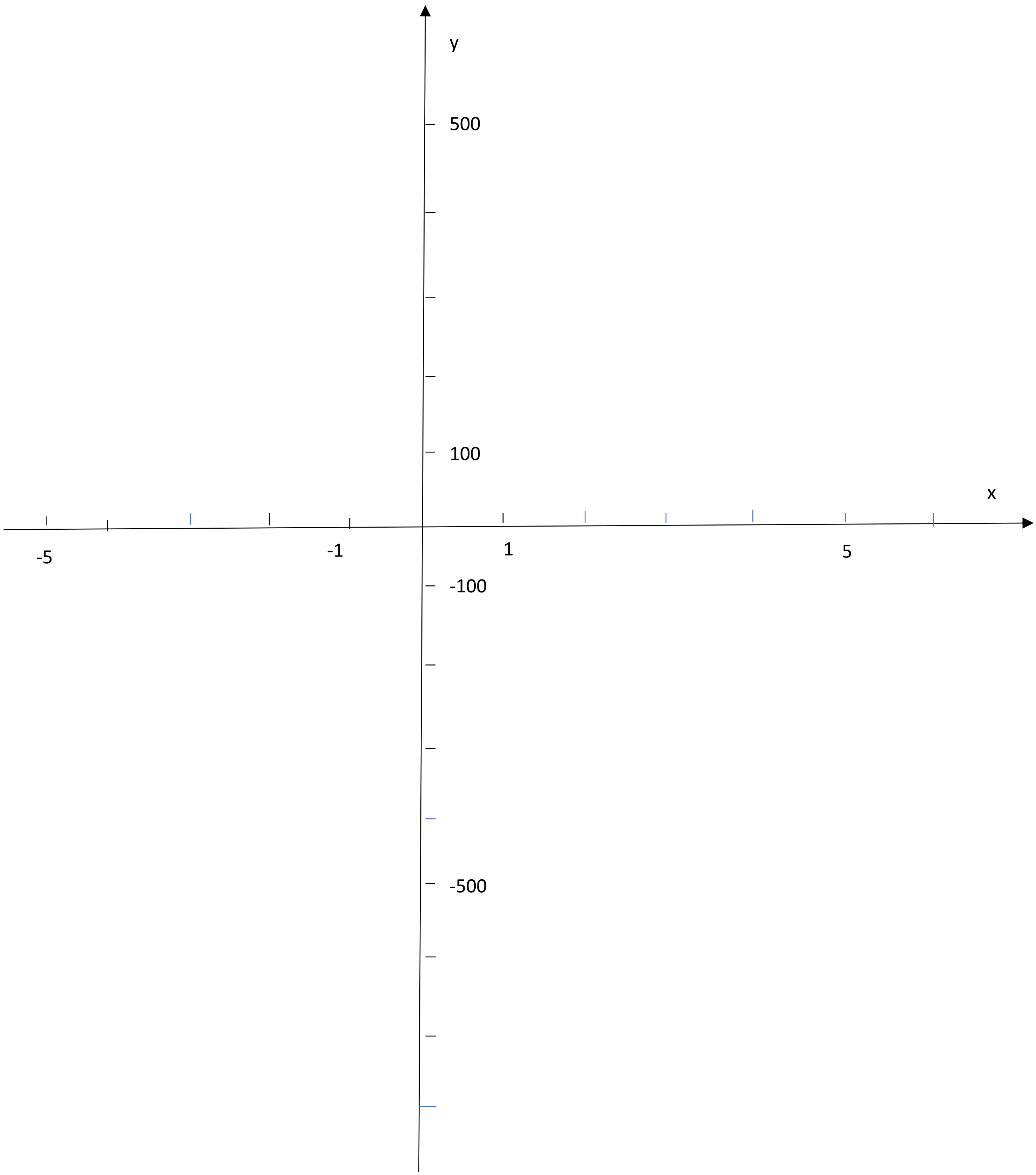
b. [2 marks] Find the intervals where f is increasing and where f is decreasing.

c. [1 mark] Find relative (local) extrema, if any.

d. [2 marks] Find the intervals where the graph of f is concave upward and where the graph of f is concave downward.

e. [1 mark] Find point(s) of inflection, if any.

f. [3 marks] Label each important point and sketch the graph of f.



Answers:

1. a. -4; b. DNE; c. DNE; d. 0; e. 0; f. x=2; g. y=0;
   h. $x = -2$, the 3rd condition fails, x=0, the 2nd condition fails, x=2 the 1st condition fails.

2. a. 4; b. 2/5

3. a. 21,176.47. and 680,000; b. ∞

4. The concentration of drug in the bloodstream eventually decreases to zero.

5. The function is discontinuous at $x = -7$, since $f(-7)$ is undefined

   and at $x = -3$ since $\lim_{x\to -3} f(x) = \text{dne}$.

6. $f'(x) = \lim_{h\to\infty} \frac{f(x+h)-f(x)}{h} = -10x$

7. a. $y' = \frac{3}{4}x^{-1/4} - 3\pi x^{\pi-1} - 4e^{-x}$; b. $f'(x) = \frac{5(x^4+3)-4x^3(5x+2)}{(x^4+3)^2}$;

   c. $g'(x) = 2\tan(3x)\sec^2(3x)3 + \cos x\cos(5x) - 5\sin x\sin(5x)$;

   d. $y' = (x^4+3)^{4x-1}\left[4\ln(x^4+3) + \frac{4x^3(4x-1)}{x^4+3}\right]$

8. a. $f''(x) = \frac{-25}{(7-5x)^2}$; b. $-\frac{25}{49}$

9. \$61,827

10. The weekly demand decreases at the rate of 125 units/week.

11. a. $R(x) = 4000x - 0.5x^2$; b. $P(x) = -0.5x^2 + 3950x - 12{,}000$;

    c. $P'(1200) = 27500$ approximates the profit from producing and selling the 1201st unit ;

    d. $x = 3950$. Maximum profit $P(3950) = 7{,}789{,}250\$$

12. a. $y' = \frac{2xy - 2y^2 + 3x^2}{4xy - x^2}$; b. $y = \frac{21}{13}x + \frac{60}{13}$

13. \$360

14. a. (0,0), (5,0), (0,0); b. f $\uparrow$ on $(-\infty, 0) \cup (4, \infty)$ and f $\downarrow$ on (0,4);

    c. (0,0) relative maximum, (4,-512) relative minimum;

    d. CD on $(-\infty, 3)$ and CU on $(3, \infty)$; e. $(3, -324)$ inflection point