DAWSON COLLEGE
DEPARTMENT OF MATHEMATICS

FINAL EXAMINATION

CALCULUS-III

May 24, 2016 **Time: 2:00 pm-5:00 pm**

Instructor: A. Panait, T. Kengatharam

Name:
ID:

Instructions:

- Translation and regular dictionaries are permitted.
- Scientific non-programmable calculators are permitted.
- Print your name and ID in the provided space.
- This examination booklet must be returned intact.

This examination consists of 20 questions. Please ensure that you have a complete examination before starting.

(1) [*5 marks*]Find a power series representation and its radius of convergence for the function

$$f(x) = \frac{2x}{(1+2x)^2}.$$

(Hint:$\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n$)

(2) [*5 marks*]Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{2^{n+2}}.$$

(Hint: You may use question (1) for a specific value of x)

(3) [*5 marks*]Approximate the sum of the convergence series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{10^n n!}$ correct to four decimal places.

(4) [*5 marks*]Evaluate the integral $\int_0^1 xe^{-x^3}\,dx$ as an infinite series. (Hint: You may use $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$)

(5) [*5 marks*]Consider the curve with parametric equations $x = e^t, y = te^{-t}$. Find $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward?

(6) [*5 marks*]Sketch the curve with polar equation $r = 1 - \cos\theta$ for $0 \leq \theta < 2\pi$.

(7) [*5 marks*] Find the equation of the tangent line to the curve with parametric equations $x = 1 + \sqrt{t}, y = e^{t^2}$ at the point $(2, e)$.

(8) [*5 marks*]Find the arc length of the curve $\underline{r}(t) = (\cos t, \sin t, \ln(\cos t))$ for $0 \leq t \leq \pi/4$.

(9) [*5 marks*]Show that the curvature of a circle with radius a is $\frac{1}{a}$.

(10) [*5 marks*]Find the equation of the osculating plane to the curve $\underline{r}(t) = (t, t^2, t^3)$ at $(1, 1, 1)$.

(11) [*5 marks*]Study the continuity of

$$f(x,y) = \begin{cases} \frac{xy\cos y}{3x^2+y^2} & \text{if} \quad (x,y) \neq (0,0) \\ \frac{1}{4} & \text{if} \quad (x,y) = (0,0) \end{cases}.$$

(12) [*5 marks*]Find the maximum of the function $f(x, y, z) = 3x + 2y + 4z$ under the constrant $g(x, y, z) = x^2 + 2y^2 + 6z^2 - 1 = 0$.

(13) [*5 marks*]Find all critical points of $f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ and classify them.

(14) [*5 marks*]If a particle with mass m moves with position vector $\underline{r}(t)$, then its angular momentum is defined by $\underline{L}(t) = m\underline{r}(t) \times \underline{v}(t)$ and its torque as $\underline{\tau}(t) = m\underline{r}(t) \times \underline{a}(t)$, where $\underline{v}(t)$ and $\underline{a}(t)$ are the particle's velocity and accelaration respectively. Show that $\underline{L}'(t) = \underline{\tau}(t)$.

(15) [*5 marks*]If $z = x\sqrt{4 + x^2y^2}$, $x = r^2 + s^2$ and $y = r^2s^2$ find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial r}$ when $r = 1$ and $s = 0$.

(16) [*5 marks*]Find the volume of the solid enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0, y = 1, y = x$ and $z = 0$.

(17) [*5 marks*]Find the volume of the solid that lies inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

(18) [*5 marks*]Compute the volume of the tetrahedron bounded by the plane $x + 2y + 3z = 6$ and the three coordinate planes.

(19) [*5 marks*]Using cylindrical coordinates evaluate $\int\int\int_E x^2 dv$ where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$ and below the cone $z^2 = 4x^2 + 4y^2$.

(20) [*5 marks*]Prove that $\int\int\int_E ze^{(x^2+y^2+z^2)^6} dV \le 0$, where E is the lower hemisphere $\{(x, y, z) \mid x^2 + y^2 + z^2 \le 1,\ z \le 0\}$.
(Hint: you may use the spherical coordinates $x = \rho \sin\phi \cos\theta,\ y = \rho \sin\phi \sin\theta,\ z = \rho \cos\phi$ for which $dV = \rho^2 \sin\phi d\rho d\phi d\theta$.)