

**Dawson College**  
**Mathematics Department**  
**Final Examination**

**201-NYB-05, Calculus II- Science**

**Sections: 01,02,03,04,05,06,07,08,09,10,11,12,17,18**

**Tuesday, May 19, 2015      14:00-17:00**

**Student Name:** \_\_\_\_\_ **Student I.D. #:** \_\_\_\_\_

**Instructors:** I. Gorelyshev, L.Frajberg, A.Hariton, T. Kengatharam, S. Muise, V. Ohanyan, A. Panait, B. Seamone (Ainhoa Aparici Monforte), S. Shahabi, S.Soltuz (O. Khalil), B. Szczepara, O.Veres, O. Zlotchevskaia

**INSTRUCTIONS:**

- Print your name and student number in the space provided above.
- Attempt all questions. Show all your work.
- All questions are to be answered directly on the examination paper.
- Only the following calculators are permitted:  
EL-531 XG or EL-531X
- Translation and regular dictionaries are permitted.
- This examination consists of 15 questions on 17 pages, including this cover page.
- Please ensure that you have a complete exam package before starting.
- The examination must be returned intact.

| Question#        | Marks<br>obtained/available |
|------------------|-----------------------------|
| 1                | /25                         |
| 2                | /5                          |
| 3                | /4                          |
| 4                | /3                          |
| 5                | /3                          |
| 6                | /8                          |
| 7                | /4                          |
| 8                | /4                          |
| 9                | /8                          |
| 10               | /4                          |
| 11               | /12                         |
| 12               | /5                          |
| 13               | /5                          |
| 14               | /5                          |
| 15               | /5                          |
| <b>Total/100</b> |                             |

1. Find or evaluate the following integrals.

a. [5 marks]  $\int \tan^3(3x) \sec^4(3x) dx$

b. [5 marks]  $\int \frac{dx}{x^2\sqrt{1-25x^2}}$

c. [5 marks]  $\int x^2 \sin(2\pi x) dx$

d. [5 marks]  $\int \frac{x-1}{(x+1)(x^2+2)} dx$

e. [5 marks]  $\int_4^{49} \frac{3}{\sqrt{\sqrt{x} + 2}} dx$

2. a. [4 marks] Using the limit of a Riemann Sum, evaluate:

$$\int_1^4 (5 - 3x^2) dx$$

- b. [1 mark] Check the correctness of your answer in a. using the Fundamental Theorem of Calculus (Evaluation Theorem).

$$\sum_{i=1}^n c = cn; \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}; \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}; \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

3. [4 marks] Find the average value of the function  $f(x) = 3 \arcsin x$  over the interval  $\left[0, \frac{1}{2}\right]$ .

4. [3 marks] If  $f$  is continuous on  $[a, b]$  show that  $\int_0^1 f[a - (a - b)x]dx = \frac{1}{b-a} \int_a^b f(x)dx$

5. [3 marks] Given  $3e + \int_c^x \frac{f(t)}{e^{2t}} dt = \ln(3x)$ ,  $x > 0$ ,  $c$  - constant

Find the function  $f(x)$ .



6. Evaluate the improper integral or show it diverges.

a. [4 marks]  $\int_0^{\infty} \frac{9 \arctan x}{1+x^2} dx$

b. [4 marks]  $\int_0^6 \frac{dx}{(x-3)^4}$

7. [4 marks] Find the arc length of the curve.

$$f(x) = \frac{1}{12} (4x^2 + 4)^{\frac{3}{2}}, \quad 0 \leq x \leq 3$$

8. [4 marks] Find the area of the region enclosed by the graphs of the following functions.  
 $y = 16 - x^2$ ,  $y = x^2 - 4x$

9. Consider the region enclosed by the curve and the line

$$y = -x^2 + 6x - 5, \quad y = x - 1$$

- a. [3 marks] Set up, but do not evaluate the integral for the volume of the solid when the given region is rotated about the  $x$ -axis;
- b. [5 marks] Find the volume of the solid when the given region is rotated about the line  $x = 6$ .

10. [4 marks] Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\left\{ \frac{\pi(2n)!}{4n^2(2n-2)!} \right\}_{n=1}^{\infty}$$

11. For each of the following series, determine whether the series converges or diverges. State which test you are using for each problem.

a. [4 marks]  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{8n^9+n^5}+4}{5+n^3}$

b. [4 marks]  $\sum_{n=1}^{\infty} \frac{n+4}{n^2 \sqrt{n}}$

c. [4 marks]  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n^3)}$

12. [5 marks] Determine whether the series converges absolutely, converges conditionally, or diverges. State which test you are using.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

13. [5 marks] Find the radius of convergence and the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{(x+4)^n}{(n+1)3^n}$$



14. [5 marks] Find the sum of the following series if it converges or show it diverges.

$$\sum_{n=1}^{\infty} \frac{4 \cdot 8^{n-1} + 11^n}{16^{n-2}}$$

15. [5 marks] Find the Maclaurin series representation of  $f(x) = \cos(3\pi x)$

Answers:

1.

a.  $\frac{1}{3} \left( \frac{\sec^6(3x)}{6} - \frac{\sec^4(3x)}{4} \right) + C$

b.  $-\frac{\sqrt{1-25x^2}}{x} + C$

c.  $-\frac{1}{2\pi} x^2 \cos(2\pi x) + \frac{1}{2\pi^2} x \sin(2\pi x) + \frac{1}{4\pi^3} \cos(2\pi x) + C$

d.  $-\frac{2}{3} \ln|x+1| + \frac{1}{3} \ln(x^2+2) + \frac{1}{3\sqrt{2}} \arctan\left(\frac{x}{\sqrt{2}}\right) + C$

e. 52

2. -48

3.  $\frac{\pi}{2} + 3\sqrt{3} - 6$

4. *proof*

5.  $f(x) = \frac{e^{2x}}{x}$

6.

a.  $\frac{9\pi^2}{8}$  *converges*

b. *diverges*

7.  $\frac{63}{3}$

8.  $72u^2$

9.

a.  $\pi \int_1^4 [(-x^2 + 6x - 5)^2 - (x - 1)^2] dx$

b.  $\frac{63\pi}{2} u^3$

10.  $\pi$  *convergent*

11.

a. *divergent, Test for Divergence*

b. *convergent, Limit Comparison Test*

c. *divergent, Integral Test*

12. *conditionally convergent*

13.  $R = 3, I = [-7, -1)$

14.  $\frac{2096}{5}$  *convergent*

15.  $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n} \pi^{2n} x^{2n}}{(2n)!}$