

DAWSON COLLEGE
Mathematics Department

ALTERNATE FINAL EXAMINATION
Calculus II, (201-MA2-DW), Winter 2024

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Student Name: _____

Student ID. Number: _____

Instructor's Name: _____

Instructions:

- Print your name and ID number in the provided space.
- Solve the problems in the space provided for each question and show all your work clearly and indicate your final answer.
- Reverse sides of pages may be used for rough work and/or to complete a solution.
- Only the calculators SHARP EL-531* are permitted.
- This examination booklet must be returned intact.
- A copy of the formula sheet and Z-table is provided for you separately.

This exam booklet consists of 15 questions onpages excluding the cover page. Please ensure that you have a complete examination booklet before starting.

1. (6 marks) In a geometric sequence $a_4 = -189$ and $a_9 = 45,927$

(a) Find the common ratio of the sequence.

(b) Find the first term of the sequence.

(c) Find the sum of its first 10 terms.

Answers. $a_1 = 7$ and $r = -3$

2. (4 marks) A bank loan of 9,000\$ is repaid in monthly instalments of 150\$ over 5 years, plus 1% interest on the unpaid balance at the start of each month. What is the total amount of interest paid?

Answer. 2655\$

3. (6 marks) Use the limit definition of definite integrals (Riemann Sum) to evaluate the integral. No marks will be given for using antiderivative rules.

$$\int_{-1}^2 (1 + 3x - x^2) dx$$

Answer. 4.5

4. (5 marks) The number of Netflix users in North America was growing at the rate of

$$R(t) = 9.294 e^{(0.027t-1)} \quad 0 \leq t \leq 10$$

million users per year where $t=0$ corresponds to 2014. If the number of users in 2014 was 47.9 million, find the number of users in 2021. (Round the answer to three decimal places.)

Answer. $C = -78.732$, $N(7) = 74.245$ millions

5. (5 marks) Find the average value of $f(t) = \frac{\sin(1/t)}{t^2}$ over the interval $\left[\frac{1}{\pi}, \frac{3}{\pi}\right]$.

Answer. $\frac{3\pi}{4}$

6. (5 marks) The daily marginal profit function associated with producing a certain device is

$$P'(x) = -0.0006x^2 + 0.04x + 40$$

where $P'(x)$ is measured in dollars per unit and x denotes the number of units manufactured. What is the additional daily profit realized if the production and sales of this device increased from 200 to 300 units per day?

Answer. 8800\$

7. (5 marks) Find the area of the region completely enclosed by the graphs of the functions $y = x^2 - 2x - 3$ and $y = -x^2 - 9x + 1$.

Answer. 30.375

8. (20 marks) Solve the three indefinite integrals and evaluate the definite integral.

(a) $\int \frac{\sqrt{x}}{3 - \sqrt{x}} dx$

(b) $\int \frac{\sec^2(4t)}{5 + 2 \tan(4t)} dt$

(c) $\int (4x + 7) \cos x dx$

(d) $\int_1^e \frac{\ln x}{x^2} dx$

Answer. Checked!

9. (6 marks) The weekly supply function for a certain model of batteries is given by

$$p = \sqrt{134 + 5x}$$

where p is the unit price in dollars and x (in units of a thousand) is the quantity supplied per week. What is the producers' surplus if the selling price is set at \$13?

Answer. $PS = \$4888$

10. (2+3 marks) A developing country's income distribution is described by the Lorenz function

$$f(x) = \frac{13}{14}x^2 + \frac{1}{14}x.$$

(a) Find $f(0.8)$ and interpret your result.

(b) Find the coefficient of inequality, or Gini Index, of the given Lorenz curve. (Round your answer to four decimal places.)

Answer.

11. (10 marks) Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} 3x \cdot \tan \frac{1}{x}$

(b) $\lim_{x \rightarrow 0} \frac{2x + 1 - e^{2x}}{5x \sin x}$

Answer. a) , b)

12. (6 marks) Evaluate the integral if it converges.

$$\int_{-\infty}^0 \frac{e^x}{3 + e^x} dx$$

Answer. $\ln(3/2)$ Convergent.

13. (6 marks) Find the general solution of

$$(x^3 + 1)y' = 6x^2y$$

and then find the particular solution of the differential equation that satisfies the condition $y(1) = 1$.

Answer. $y = C(x^2 + 1)^2$, $y = \frac{1}{4}(x^2 + 1)^2$

14. (5 marks) The life span (in years) of a certain battery for a certain electric car is continuous random variable with probability density function

$$f(x) = 25(25 + x^2)^{-3/2} \quad 0 < x < \infty.$$

How long is one of these batteries expect to last?

Answer. $E(X) = 4$

15. (6 marks) The medical records of infants delivered at a certain hospital in Montreal show that the infants' lengths at birth in cm are normally distributed with a mean of 51 cm and standard deviation of 6.5 cm. Find the probability that an infant selected at random from among those delivered at the hospital measures:

(a) more than 56 cm.

(b) between 48.5 cm and 54.5 cm.