Dawson College

Physics 203-NYC-05 Waves, Optics & Modern Physics

Sample Final Examination

This exam is divided into two parts:

Part I: Problems (*10 marks each*) Solve all six problems. Show all of your work, clearly and in order, to receive full marks. If you use a formula not given on the formula sheet, a derivation must be shown.

Part II: Multiple Choice Questions (*2 marks each*) Answer all twenty questions. Circle the best response from the choices given. If your final selection is unclear you will not be given the marks. No marks will be awarded for diagrams, calculations, or reasoning.

Additional instructions:

1. The time allotted for this examination is three hours.
2. Answer directly on the question sheet. If you need extra room, you may use the back of another page.
3. Use $v_{\text{sound}} = 343\,\text{m/s}$.
4. When finished, return this entire package and the formula sheet to your instructor.

Good luck! ☺

P1	P2	P3	P4	P5	P6	MC	Total /100

Part I: Problems (*10 marks each*)

Solve all six problems. Show all of your work, clearly and in order, to receive full marks. If you use a formula not given on the formula sheet, a derivation must be shown.

1. A transverse wave with an amplitude of 2.00 cm travels down a string under 50.0 N of tension. The distance between one wave crest and the next is 70.0 cm. A fixed point on the string oscillates 200 cycles in 12.0 s.

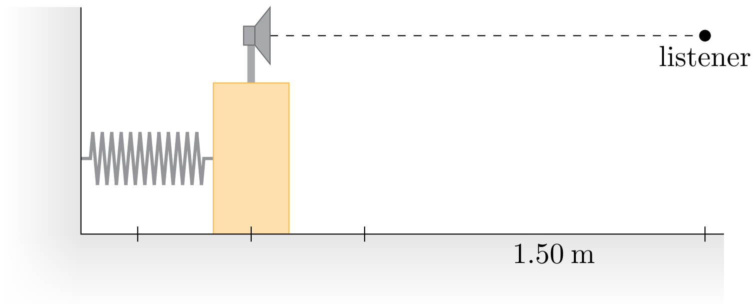
    (a) What are the wave speed and the maximum transverse speed of a point on the string?

    (b) Explain the difference between the speeds found in part (a).

    (c) What is the mass-per-unit-length of the string?

    (d) What is the power transmitted along the string by these oscillations?

2. A block with a sound source oscillates along a horizontal frictionless surface with an amplitude of 50.0 cm and a maximum speed of 20.0 m/s.

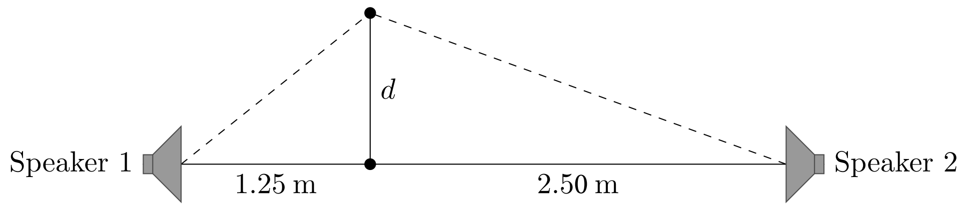


(a) If the spring constant is 600 N/m, what is the mass of the block (including the sound source)?

(b) If the sound source frequency is 440 Hz (at rest), waht are the lowest and the highest frequencies heard by the listener?

(c) If the closest the block comes to the listener is 1.50 m and the maximum sound intensity from the block's sound source at that instant is 60.0 dB, what is the minimum sound intensity heard by the listener?

3. Two speakers emit equal amplitude waves with a frequency of 343 Hz. A listener is initially standing 1.25 m from speaker 1 and 2.50 m from speaker 2 on a line connecting the two sources.



(a) What is the phase difference at the initial position of the listener? Does this correspond to maximum constructive, maximum destructive, or other interference?

(b) If the listener were to walk along a line perpendicular to the line connecting the two speakers, what would they next experience? Maximum constructive or maximum destructive interference? Defend your answer.

(c) Find the distance d that corresponds to the first maximum constructive or maximum destructive interference case found in part b.

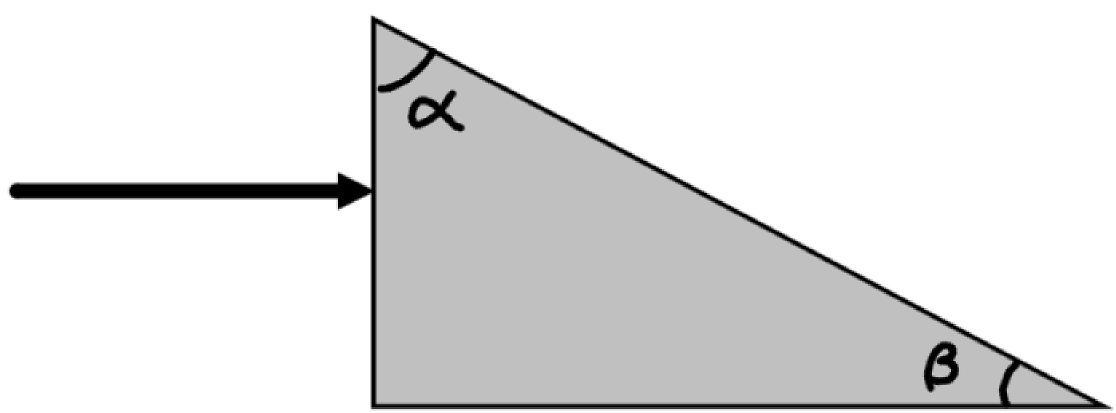
4. White light strikes a prism at right angles (90^o) to the left face – as shown below. The prism is made of plastic that has indices of refraction that change slightly depending on the colour of light. If air surrounds the prism, α=50.00^o and β=40.00^o, and the indices are n_{700nm}=1.516 and n_{400nm}=1.538, then answer the following:

a) At what angle of incidence does the light enter the prism?

b) For incident light that has wavelength 700nm, what is the speed and frequency of the corresponding light as it moves through the prism?

c) In the diagram below, draw a complete trace of both red light (700.0nm) and violet light (400.0nm) as it refracts/reflects through the prism and as it exits into the air. In your drawing, you can make your angles approximate, but take care to show if/where the red light and the violet light separates into distinct directions.

d) Find the angle(s) of the red light (700.0nm) and violet light (400.0nm) as each exits the prism.



5. Hydrogen emits visible light at specific wavelengths. The wavelengths are traditionally named as follows: $H_\alpha = 656.3$ nm (red), $H_\beta = 486.1$ nm (cyan), $H_\gamma = 434.1$ nm (blue), $H_\delta = 410.2$ nm (violet). Light from hydrogen passes through a diffraction grating creating an emission spectrum of fringes.

   (a) If the second-order H_α line is observed at $41.018°$, what is the number of lines per millimetre of the grating?

   (b) What is the angular separation of the H_γ and H_δ lines in the 1st order? [Keep at least 3 decimal places in your work.]

   (c) How many complete orders are visible on one side of the centre of the spectrum?

   (d) Are there any overlapping orders? If yes, which ones?

6. A by-product of some nuclear power reactors is the isotope ^{239}Pu. This isotope has a half-life of 24120 years and decays by alpha emission: $^{239}\text{Pu} \rightarrow {}^{235}\text{U} + \alpha$. Suppose a sample of this plutonium isotope has a mass of 0.7939109 kg at $t = 0$.

   (a) How many atoms are initially in the sample?

   (b) What is the initial activity of the sample in Becquerels?

   (c) How many years does it take for the activity to decrease to 0.100 Bq? [Use the conversion factor: 1 year = 365.25 days]

Part II: Multiple Choice Questions (*2 marks each*)

Answer all twenty questions. Circle the best response from the choices given. If your final selection is unclear you will not be given the marks. No marks will be awarded for diagrams, calculations, or reasoning.

1. A mass at the end of an ideal spring vibrates with period T. If you double the mass, how must you change the force constant of the spring to achieve a period of $2T$?

   (a) Decrease it by a factor of 4.

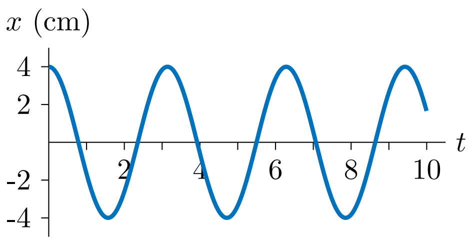
   (b) Increase it by a factor of $\sqrt{2}$.

   (c) Decrease it by a factor of $\sqrt{2}$.

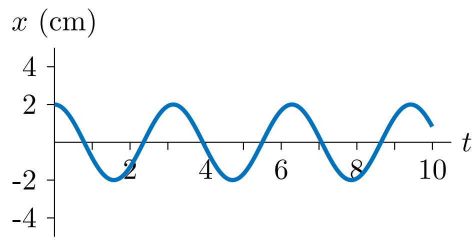
   (d) Decrease it by a factor of 2.

   (e) Increase it by a factor of 2.

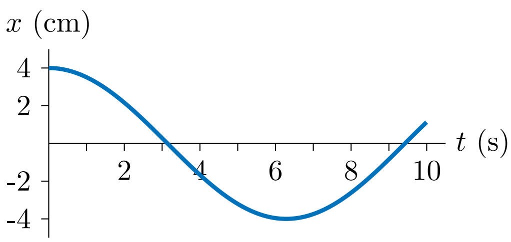
2. Which of following graphs represents simple harmonic motion with an amplitude of 4 cm and an angular frequency of 2 rad/s?



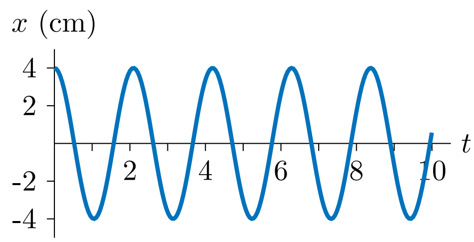
(a)



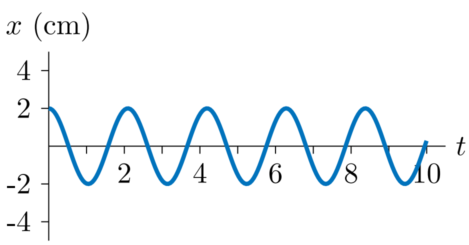
(b)



(c)



(d)



(e)

3. Which of the following is a *false* statement?

   (a) The speed of a wave and the speed of the vibrating particles that constitute the wave are different entities.

   (b) For a transverse wave the particle motion is perpendicular to the direction of propagation of the wave.

   (c) A wave in which the particle motion is parallel to the direction of propagation of the wave is called a longitudinal wave.

   (d) Waves transport energy and matter from one region to another.

   (e) Not all waves are mechanical waves.

4. Four separate travelling waves are described by the following equations, where $y(x,t)$ represents the displacement:

$$
\begin{aligned}
\text{I:} \quad & y(x,t) = (0.12\,\text{m})\cos\left[(3.0\,\text{rad/m})x + (2.0\,\text{rad/s})t\right] \\
\text{II:} \quad & y(x,t) = (0.15\,\text{m})\sin\left[(6.0\,\text{rad/m})x - (3.0\,\text{rad/s})t\right] \\
\text{III:} \quad & y(x,t) = (0.23\,\text{m})\cos\left[(3.0\,\text{rad/m})x + (6.0\,\text{rad/s})t\right] \\
\text{IV:} \quad & y(x,t) = (0.29\,\text{m})\sin\left[(1.5\,\text{rad/m})x + (1.0\,\text{rad/s})t\right]
\end{aligned}
$$

   Which two waves have the same wave speed?

   (a) I and III

   (b) I and II

   (c) I and IV

   (d) III and IV

   (e) II and III

5. An open-open tube 0.46 m long supports a standing wave of frequency 1114 Hz. What is the distance from the centre of the tube to the nearest antinode?

   (a) 0.15 m

   (b) 0.12 m

   (c) 0.077 m

   (d) 0.038 m

   (e) There is an antinode exactly at the centre of the tube.

6. A 137 cm long wire that is fixed at both ends has a fundamental frequency of 87.5 Hz. The tension in the wire is then changed so that the fundamental frequency becomes 175.0 Hz. What is the ratio of the new fundamental wavelength to the old fundamental wavelength, $\lambda_{\text{new}}/\lambda_{\text{old}}$?

   (a) 1
   (b) $\sqrt{2}$
   (c) 2
   (d) $1/\sqrt{2}$
   (e) 1/2

7. A car is on a road parallel to a railroad track and right beside it. The car is travelling east at 30.0 m/s while a train travelling west at 50.0 m/s is approaching it. If the car honks its horn at a frequency of 1.00 kHz, what is the wavelength of the sound perceived by a person riding in the train?

   (a) 0.235 m
   (b) 0.275 m
   (c) 0.294 m
   (d) 0.313 m
   (e) 0.344 m

8. If the distance from a point source of sound is tripled, the intensity will be what multiple of its original value?

   (a) 3
   (b) 1/3
   (c) 1
   (d) 9
   (e) 1/9

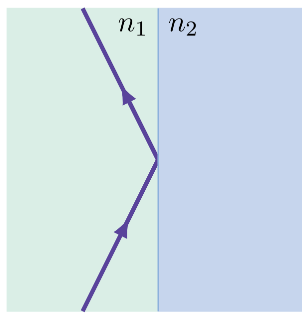
9. At a distance of 2.00 m from a point source of sound, the sound intensity level is 80.0 dB. At a distance of 4.00 m from the source, what is the sound intensity level?

   (a) 74.0 dB
   (b) 77.0 dB
   (c) 60.0 dB
   (d) 20.0 dB
   (e) 40.0 dB

10. Which one of the following is a correct ranking of electromagnetic waves from longest wavelength to shortest wavelength?

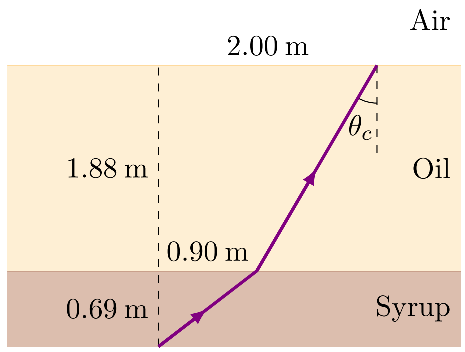
   (a) radio waves, infrared, microwaves, UV, visible, X-rays, gamma rays
   (b) radio waves, microwaves, visible, X-rays, infrared, UV, gamma rays
   (c) radio waves, infrared, X-rays, microwaves, UV, visible, gamma rays
   (d) radio waves, UV, X-rays, microwaves, infrared, visible, gamma rays
   (e) radio waves, microwaves, infrared, visible, UV, X-rays, gamma rays

11. A ray of light strikes a boundary between two transparent materials and there is no transmitted ray, as shown in the figure. What can you conclude about the indices of refraction of these two materials?



   (a) $n_1 \geq n_2$
   (b) $n_1 > n_2$
   (c) $n_1 = n_2$
   (d) $n_1 < n_2$
   (e) $n_1 \leq n_2$

12. A tank holds a layer of oil, 1.88 m thick, which floats on a layer of syrup that is 0.69 m thick. Both liquids are transparent and do not intermix. A ray of light originates from the bottom of the tank and crosses the oil-syrup interface a horizontal distance of 0.90 m its starting point. The ray continues and arrives at the oil-air interface a horizontal distance of 2.00 m from the starting point. The angle of incidence at the oil-air interface is the critical angle. What is the index of refraction of the oil?



(a) $n_{\text{oil}} = 2.02$

(b) $n_{\text{oil}} = 1.94$

(c) $n_{\text{oil}} = 2.00$

(d) $n_{\text{oil}} = 1.96$

(e) $n_{\text{oil}} = 1.98$

13. In a double-slit experiment, if the slit separation is increased, which of the following happens to the interference pattern shown on the screen?

(a) The minima and maxima stay at the same positions.

(b) The maxima stay at the same positions.

(c) The minima stay at the same positions.

(d) The minima get closer together.

(e) The maxima get further apart.

14. Light passes through a pair of very thin parallel slits. The resulting interference pattern is viewed far from the slits at various angles θ relative to the centre-line coming outward from the midpoint between the slits. The central bright fringe is at $\theta = 0°$, and the next bright fringes on either side of the centre are at $\theta = \pm 15°$. If the central bright fringe has an intensity I_0, what is the intensity of the next bright fringe on either side of it?

(a) I_0

(b) $I_0 \cos(15°)$

(c) $I_0 \cos^2(15°)$

(d) $I_0/\sqrt{2}$

(e) $I_0/2$

15. A diffraction grating has 450 lines per mm. What is the highest order that contains the entire visible spectrum from 400 nm to 700 nm?

    (a) $m = 2$
    (b) $m = 3$
    (c) $m = 4$
    (d) $m = 5$
    (e) $m = 6$

16. If the diameter of a radar dish is doubled, what happens to its resolving power assuming that all other factors remain unchanged? [Use the small angle approximation]

    (a) The resolving power is reduced to 1/4 of its original value.
    (b) The resolving power is reduced to 1/2 of its original value.
    (c) The resolving power doubles.
    (d) The resolving power quadruples.
    (e) The resolving power does not change unless the focal length changes.

17. A laser pulse of duration 25 ms has a total energy of 1.2 J. If the wavelength of the light is 463 nm, how many photons are emitted in that one pulse?

    (a) 3.4×10^{19} photons
    (b) 1.1×10^{17} photons
    (c) 2.2×10^{17} photons
    (d) 6.9×10^{19} photons
    (e) 2.8×10^{18} photons

18. Monochromatic light strikes a metal surface and electrons are ejected from the metal. If the intensity of the light is increased, what will happen to the ejection rate and maximum energy of the electrons?

    (a) Same ejection rate; same maximum energy
    (b) Greater ejection rate; same maximum energy
    (c) Same ejection rate; greater maximum energy
    (d) Greater ejection rate; greater maximum energy
    (e) Electrons would no longer be ejected

19. When a blackbody has a temperature T, the peak intensity of its radiation is at wavelength λ. If the blackbody has a temperature $2T$, what is the wavelength of the peak intensity?

    (a) 16λ
    (b) $\lambda/2$
    (c) λ
    (d) $\lambda/16$
    (e) 2λ

20. The hydrogen emission wavelength $434.0\,\text{nm}$ is produced by the transition to the $n = 2$ energy level from which energy level?

    (a) $n = 3$
    (b) $n = 4$
    (c) $n = 5$
    (d) $n = 6$
    (e) $n = \infty$

Answers

Problems

1. (a) $v = 11.7$ m/s, $v_{y,\max} = 2.09$ m/s

   (b) The wave speed is a property of the medium while the maximum transverse speed is determined by the source. These speeds are independent of each other.

   (c) $\mu = 0.367$ kg/m

   (d) $P = 9.40$ W

2. (a) m = 0.375 kg

   (b) $f_{\text{high}} = 467$ Hz, $f_{\text{low}} = 416$ Hz

   (b) $\beta_2 = 55.6$ dB

3. (a) $\Delta\phi = 2.5\pi$ rad (somewhere in between)

   (b) MCI

   (c) $d = 1.36$ m

4. (a) $\theta_2 = 0^\circ$

   (b) v = 41.98×10^8 m/s, $f = 4.29 \times 10^{14}$ Hz

   (c) See full solutions.

   (d) $\theta_{4\text{ red}} = 15.3^\circ$, $\theta_{4\text{ violet}} = 15.5^\circ$

5. (a) 500 lines/mm

   (b) $\Delta\theta = 0.700^\circ$

   (c) 3 complete orders

   (d) $m = 2$ and $m = 3$

6. (a) $N_0 = 2.00 \times 10^{24}$ atoms

   (b) $R_0 = 1.82 \times 10^{12}$ Bq

   (c) $t = 1.06 \times 10^6$ years

   (d) $E = 1.68 \times 10^{12}$ J

Multiple Choice

1. (d)	5. (c)	9. (a)	13. (d)	17. (e)
2. (a)	6. (a)	10. (e)	14. (a)	18. (b)
3. (d)	7. (d)	11. (b)	15. (b)	19. (b)
4. (c)	8. (e)	12. (e)	16. (c)	20. (c)

Solutions

Problems

1. A transverse wave with an amplitude of 2.00 cm travels down a string under 50.0 N of tension. The distance between one wave crest and the next is 70.0 cm. A fixed point on the string oscillates 200 cycles in 12.0 s.

   (a) What are the wave speed and the maximum transverse speed of a point on the string?

   (b) Explain the difference between the speeds found in part (a).

   (c) What is the mass-per-unit-length of the string?

   (d) What is the power transmitted along the string by these oscillations?

given:

$A = 2.00\,\text{cm}$, $T = 50.0\,\text{N}$, $\lambda = 70.0\,\text{cm}$, $f = \frac{200}{12.0} = 16.\dot{6}\,\text{Hz}$

a) $V_{wave} = \lambda f = (70.0)(16.\dot{6}) = 1167\,\text{cm/s} = \boxed{11.7\,\text{m/s}}$

$V_{trans,max} = A\omega = A(2\pi f) = (2.00)(2\pi)(16.\dot{6}) = 209.4\,\frac{\text{cm}}{\text{s}} = \boxed{2.09\,\text{m/s}}$

b) V_{wave} is the speed of the disturbance or the crests of the wave and it depends only on the medium - not the wave.

$V_{trans,max}$ is the maximum speed of the medium over any cycle and it depends on the wave, not the medium.

Note: if the question asked about <u>velocity</u> then it would be important to add that the direction of V_{wave} and $V_{trans,max}$ are perpendicular to each other.

c) $V_{wave} = \lambda f$ and $V_{wave} = \sqrt{\dfrac{T}{\mu}}$ (for a string)

$\therefore \quad \mu = \dfrac{T}{\lambda^2 f^2} = \dfrac{50.0\,N}{(0.70\,m)^2(16.\dot{6}\,s^{-1})^2} = \boxed{0.367\,\frac{kg}{m}}$

↳ since $N = \frac{kg\,m}{s^2}$ we need to convert λ to m

d) $P = \frac{1}{2}\mu V_{wave}\,\omega^2 A^2 = \frac{1}{2}\mu V_{wave}(2\pi f)^2 A^2$

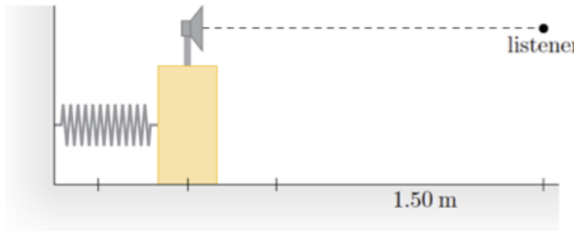
↳ don't use V_{trans} here!

$P = \frac{1}{2}\left(0.3673\,\frac{kg}{m}\right)\left(11.67\,\frac{m}{s}\right)4\pi^2(16.\dot{6}\,s^{-1})^2(0.020\,m)^2$

$= \boxed{9.40\,W}$

$1\,W = 1\,J/s = 1\,\frac{kg\,m^2}{s^3}$

2. A block with a sound source oscillates along a horizontal frictionless surface with an amplitude of 50.0cm and a maximum speed of 20.0m/s.



a) If the spring constant is 600N/m, what is the mass of the block (including the sound source)?

b) If the sound source frequency is 440Hz (at rest), what are the lowest and the highest frequencies heard by the listener?

c) If the closest the block comes to the listener is 1.50m and the maximum sound intensity from the block's sound source at that instant is 60.0dB, what is the minimum sound intensity level heard by the listener?

a) $V_{max} = A\omega$ and $\omega = \sqrt{\frac{k}{m}}$

$\therefore \quad V_{max}^2 = \frac{A^2 k}{m}$

or $\quad m = \frac{A^2 k}{V_{max}^2} = \frac{(0.50\,m)^2 (600\,N/m)}{(20\,m/s)^2} = \boxed{0.375\,kg}$

Check units: $\frac{m^2 \cdot N/m}{m^2/s^2} = \frac{(kg\,m/s^2)/m}{/s^2} = kg$ ✓

b) The highest frequency will occur when speaker is moving towards the listener with max. speed:

$f_{Highest} = 440\,Hz \left(\frac{343}{343 - 20}\right) = \boxed{467\,Hz}$

The lowest freq will occur when the speaker is moving away at max. speed:

$$f_{lowest} = 440\text{Hz}\left(\frac{343}{343+20}\right) = \boxed{416\text{ Hz}}$$

c) at $r = 1.50\text{m}$ from the listener:

$$\beta = 60\text{dB} = 10\log\left(\frac{I_{1.5m}}{1\times10^{-12}}\right)$$

$$\therefore I_{1.5m} = \left[\log^{-1}\left(\frac{60}{10}\right)\right](1\times10^{-12}) = 1\times10^{-6}\text{ W/m}^2$$

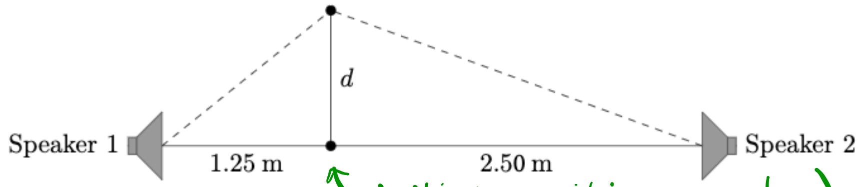
$\therefore$ at $r = 2.50\text{m}$ $(1.5\text{m} + 2A)$

$$\frac{I_{1.5m}}{I_{2.5m}} = \frac{(2.5\text{m})^2}{(1.5\text{m})^2}$$

$$\therefore I_{2.5m} = \left(1\times10^{-6}\text{ W/m}^2\right)\left(\frac{1.5^2}{2.5^2}\right) = 0.36\times10^{-6}\text{ W/m}^2$$

$$\therefore \beta_{2.5m} = 10\log\left(\frac{0.36\times10^{-6}}{1\times10^{-12}}\right) = \boxed{55.6\text{ dB}}$$

3. Two speakers emit equal amplitude waves with a frequency of 343 Hz. A listener is initially standing 1.25 m from speaker 1 and 2.50 m from speaker 2 on a line connecting the two sources.



(a) What is the phase difference at the initial position of the listener? Does this correspond to maximum constructive, maximum destructive, or other interference?

(b) If the listener were to walk along a line perpendicular to the line connecting the two speakers, what would they next experience? Maximum constructive or maximum destructive interference? Defend your answer.

(c) Find the distance d that corresponds to the first maximum constructive or maximum destructive interference case found in part b.

given: $f = 343\ Hz$

assume: $V_{sound} = 343\ m/s$

$\therefore \lambda = \frac{v}{f} = 1.00\ m$

and $\Delta\phi_{sources} = 0$

a) $\Delta\phi = \frac{2\pi}{\lambda}\Delta r + \Delta\phi_{sources}$

$= \frac{2\pi}{1.00}(2.50 - 1.25) + 0$ ($\Delta r = 1.25\ m$)

$= 2.50\pi + 0 = 2.50\pi$

get max. const. int. when $\Delta\phi = 0, 2\pi, 4\pi \ldots$

get complete dest. int when $\Delta\phi = \pi, 3\pi, \ldots$

$\therefore$ The sound waves are neither max. const. nor max. dest. int. but are instead in between.

b) to get destructive interference, need $\Delta\phi = \pi, 3\pi, 5\pi, 7\pi \ldots$ ($\Delta r = \frac{1}{2}\lambda, \frac{3}{2}\lambda \ldots$)

and for const. int. $\Delta\phi = 0, 2\pi, 4\pi \ldots$ ($\Delta r = 0, \lambda, 2\lambda \ldots$)

and since $\Delta r = 1.25\lambda$ from part a), the next possible value of Δr corresponds to

$\Delta r = \lambda \longrightarrow$ constructive

or $\Delta r = 1.5\lambda \longrightarrow$ destructive

to figure out which one it is, imagine what happens as $d \to \infty$:

As $d \to \infty$ $\Delta r \to 0$ $\therefore$ Δr decreases as the listener walks

So, as the listener walks, they would next experience **constructive interference.**

c) Now find d that corresponds to $\Delta r = \lambda = 1.00\text{m}$

$$\Delta r = \lambda = \sqrt{2.50^2 + d^2} - \sqrt{1.25^2 + d^2} = 1.00\text{m}$$

($\lambda \to 1.00\text{m}$)

now, we solve for d.

First we rearrange:

$$1.00 + \sqrt{1.25^2 + d^2} = \sqrt{2.50^2 + d^2}$$

then we square both sides:

$$1.00^2 + \left(\sqrt{1.25^2 + d^2}\right)^2 + 2(1)\sqrt{1.25^2 + d^2} = \left(\sqrt{2.50^2 + d^2}\right)^2$$

$$1.00^2 + 1.25^2 + d^2 + 2\sqrt{1.25^2 + d^2} = 2.50^2 + d^2$$

Finally, we collect, rearrange and solve:

$$2.5625 + 2\sqrt{1.25^2 + d^2} = 6.25$$

$$1.25^2 + d^2 = (1.84375)^2$$

$$\boxed{d = 1.36\text{ m}}$$

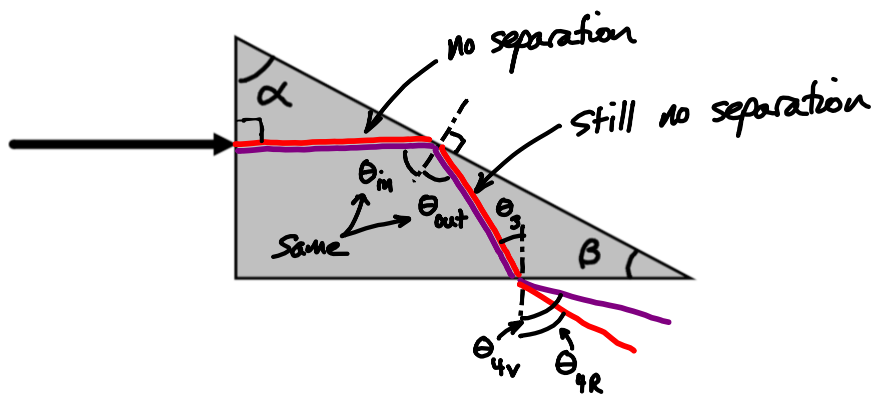
4. White light strikes a prism at right angles (90°) to the left face – as shown below. The prism is made of plastic that has indices of refraction that change slightly depending on the colour of light. If air surrounds the prism, α=50.00° and β=40.00°, and the indices are n_{700nm}=1.516 and n_{400nm}=1.538, then answer the following:

a) At what angle of incidence does the light enter the prism?

b) For incident light that has wavelength 700nm, what is the speed and frequency of the corresponding light as it moves through the prism?

c) In the diagram below, draw a complete trace of both red light (700.0nm) and violet light (400.0nm) as it refracts/reflects through the prism and as it exits into the air. In your drawing, you can make your angles approximate, but take care to show if/where the red light and the violet light separates into distinct directions.

d) Find the angle(s) of the red light (700.0nm) and violet light (400.0nm) as each exits the prism.



a) The incoming light is a 0° to the normal therefore light enters at 0° to the normal:

$$n_1 \sin \theta_1^{\,0} = n_2 \sin \theta_2$$

$$\therefore \sin\theta_2 = 0 \quad \text{or} \quad \boxed{\theta_2 = 0}$$

b) $$n_2 = \frac{c}{v_2} \quad \therefore v_2 = \frac{3.00\times10^8 \text{ m/s}}{1.516} = 1.98\times10^8 \text{ m/s}$$

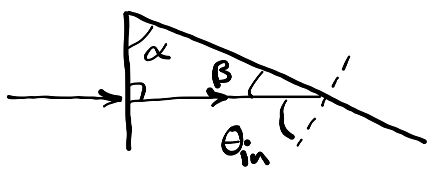
and $f_2 = f_1$ (frequency stays same)

$$\therefore f_2 = f_1 = \frac{c}{\lambda_1} = \frac{3.00\times10^8 \text{ m/s}}{700\times10^{-9} \text{ m}} = 4.29\times10^{14} \text{ Hz}$$

Note: we could have used $f_2 = \frac{v_2}{\lambda_2}$ and $\lambda_2 = n_2\lambda_1$ (answer would have been same) but using f_1 is a little faster

c) see above

d) First note that $\theta_{in} = \alpha = \theta_{out}$

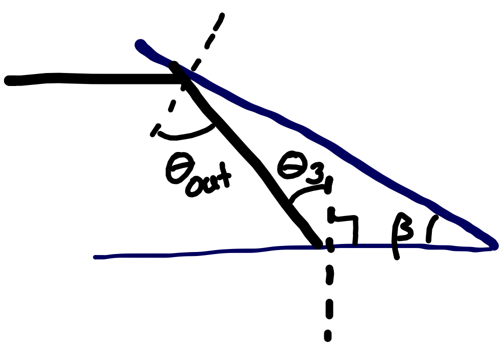


$\theta_{in} + \beta = 90$

and $\alpha + \beta = 90$

$\therefore \theta_{in} = \cancel{90} - \cancel{90} + \alpha$

Then note that



$(\cancel{90} - \theta_{out}) + (\theta_3 + \cancel{90}) + \beta = \cancel{180°}$

$\therefore \theta_3 = \theta_{out} - \beta = \alpha - \beta$

$\theta_3 = 50.0° - 40.0° = 10.0°$

and this is the same for red and violet.

$\therefore$ for red light:

$\cancel{n_{air}}^{1.00} \sin\theta_4 = \cancel{n_{700}}^{1.516} \sin 10°$

$\therefore \theta_4 = \sin^{-1}(1.516 \sin 10°) = \boxed{15.3°}$

and for violet

$\theta_{4v} = \sin^{-1}(1.538 \sin 10°) = \boxed{15.5°}$

5. Hydrogen emits visible light at specific wavelengths. The wavelengths are traditionally named as follows: H_α = 656.3 nm (red), H_β = 486.1 nm (cyan), H_γ = 434.1 nm (blue), H_δ = 410.2 nm (violet). Light from hydrogen passes through a diffraction grating creating an emission spectrum of fringes.

(a) If the second-order H_α line is observed at 41.018°, what is the number of lines per millimetre of the grating?

(b) What is the angular separation of the H_γ and H_δ lines in the 1st order? [Keep at least 3 decimal places in your work.]

(c) How many complete orders are visible on one side of the centre of the spectrum?

(d) Are there any overlapping orders? If yes, which ones?

a) $d \sin\theta = m\lambda$ for a diffraction grating

$$\therefore d = \frac{(2)\,656.3\text{ nm}}{\sin 41.018°} = 2000.0\text{ nm}$$

$$\therefore N = \frac{1}{d} = 500{,}000 \text{ lines/m} = \boxed{500 \text{ lines/mm}}$$

b) for H_γ: $d \sin\theta = m\lambda$ ($d \to 2000\times10^{-9}$ m, $m \to 1$, $\lambda \to 434.1$)

$$\therefore \theta_{1\gamma} = \sin^{-1}\left(\frac{(1)(434.1)}{2000}\right) = 12.5358°$$

and for H_δ: $\theta_{1\delta} = \sin^{-1}\left(\frac{(1)\,410.2}{2000}\right) = 11.8353°$

$$\therefore \Delta\theta = 12.536° - 11.835° = \boxed{0.700°}$$

c) For complete orders use highest λ : $H_\alpha = 656.3\,nm$

$\therefore \quad d \sin 90° = m_{max} \lambda_{max}$

$\therefore \quad m_{max} = \dfrac{d}{\lambda_{max}} = \dfrac{2000\,nm}{656.3} = 3.047$

$\therefore$ **3 complete orders are visible**

d) 1st order : $\theta_{min} = \sin^{-1}\left(\dfrac{(1)656.3}{2000}\right) = 11.83°$

$\theta_{max} = \sin^{-1}\left(\dfrac{(1)410.2}{2000}\right) = 19.16°$ — no overlap

2nd order : $\theta_{min,2} = 24.21°$

$\theta_{max,2} = 41.02°$ — These overlap

3rd order : $\theta_{min,3} = 37.97°$

$\theta_{max,3} = 79.88°$ — These overlap.

4th order : $\theta_{min,4} = 55.12°$

$\theta_{max,4} \rightarrow$ does not exist.

$\therefore$ **There are 2 overlapping orders : $n = 2$ and 3 as well as $n = 3$ and 4.**

6. A by-product of some nuclear power reactors is the isotope ^{239}Pu. This isotope has a half-life of 24120 years and decays by alpha emission: $^{239}\text{Pu} \rightarrow {}^{235}\text{U} + \alpha$. Suppose a sample of this plutonium isotope has a mass of 0.7939109 kg at $t = 0$.

(a) How many atoms are initially in the sample?

(b) What is the initial activity of the sample in Becquerels?

(c) How many years does it take for the activity to decrease to 0.100 Bq? [Use the conversion factor: 1 year = 365.25 days]

(d) What the total energy released between $t = 0$ and the time found in part (c)? [Use the masses: $m_{\text{Pu}} = 239.052157\,\text{u}$, $m_{\text{U}} = 235.043924\,\text{u}$, $m_\alpha = 4.002602\,\text{u}$]

From Formula Sheet:

$B = [Zm_H + Nm_n - m_{atom}]\,c^2$

$\frac{dN}{dt} = -\frac{1}{\tau}N,\quad N = N_0 e^{-t/\tau},\quad N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$

$r = \frac{1}{\tau} = \frac{\ln 2}{t_{1/2}}$

$\frac{dR}{dt} = -\frac{1}{\tau}R,\quad R = R_0 e^{-t/\tau},\quad R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$

$R = -\frac{dN}{dt} = rN,\quad R_0 = rN_0$

Constants

u = 1.660 539×10^{-27} kg = 931.494 MeV/c^2

m_p = 1.673×10^{-27} kg = 1.007 276 u

m_n = 1.675×10^{-27} kg = 1.008 665 u

m_e = 9.11×10^{-31} kg = 0.000 549 u

m_H = 1.007 825 u

1Ci = 3.7×10^{10} Bq = 3.7×10^{10} decay/s

a) $m_{Pu} = 239.052157\,u$

$= 3.969554 \times 10^{-25}\ \text{kg/Pu}$

$\therefore\ N_0 = \frac{0.7939109\ \text{kg}}{3.969554 \times 10^{-25}\ \text{kg/Pu}}$

$= 2.00000 \times 10^{24}\ \text{Pu}$

b) $R_0 = rN_0$

where $r = \frac{\ln 2}{t_{1/2}}$

$\therefore\ r = \frac{\ln 2}{24120\,\text{yrs}\left(\frac{365.25\,d}{yr}\right)\left(\frac{24\,h}{d}\right)\left(\frac{3600\,s}{h}\right)}$

$= 0.9106347 \times 10^{-12}$

$\therefore\ R_0 = 1.82127 \times 10^{12}\ \text{decays/s} = 1.82127 \times 10^{12}\ \text{Bq}$

c) $R = R_o \left(\frac{1}{2}\right)^{t/t_{1/2}}$

$\therefore\ 0.100 = 1.821 \times 10^{12} \left(\frac{1}{2}\right)^{t/24120}$

$\therefore\ 44.05 = \frac{t}{24120\,\text{yrs}}$

or $t = \boxed{1.06 \times 10^6 \text{ years}}$

d) Energy per decay $= \left[m_{Pu} - m_{U} - m_{\alpha}\right] c^2$

$= \left[239.052157u - 235.043924u - 4.002602u\right] c^2$

$= \left[0.005635u\right] c^2 = 0.005635u \left(931.494\ \text{MeV}/c^2\right) c^2$

$= 5.24897\ \text{MeV/decay}$

$= \left(5.24897 \times 10^6\,\text{eV}\right)\left(1.602 \times 10^{-19}\,\text{J}\right)$

$= 0.8409 \times 10^{-12}\ \text{J/decay}$

Think: Since the vast majority of the nuclei would have decayed after a million years we can say that $N_{decayed} = N_o$

$\therefore$ Total $E = \left(2.000 \times 10^{24}\ \text{decays}\right)\left(0.8409 \times 10^{-12}\ \text{J/decay}\right)$

$= \boxed{1.682 \times 10^{12}\ \text{J}}$

Multiple Choice

1. Since $T = 2\pi\sqrt{m/k}$ for a mass on a spring, a doubling of the mass must be joined by a halving of the spring constant. The answer is (d).

2. Only graphs (a), (c), and (d) have the 4 cm amplitude. An angular frequency of 2 rad/s corresponds to a period $T = 2\pi/\omega = \pi$ s. Only graph (a) has the proper period. The answer is (a).

3. Waves transport energy, but they do *not* transport material. The answer is (d).

4. In each equation the argument of the function is $kx \pm \omega t$. The wave speed $v = \lambda f = \omega/k$. The values of wave speed are $v_{\rm I} = 0.67$ m/s, $v_{\rm II} = 0.5$ m/s, $v_{\rm III} = 2$ m/s, and $v_{\rm IV} = 0.67$ m/s. The answer is (c).

5. The wavelength of the sound $\lambda = v/f = 0.308$ m. Using the wavelength and length of the tube we can find the mode number $m = 2L/\lambda = 3$. For the $m = 3$ mode there is a node at the centre of the tube. The nearest antinode is $\lambda/4 = 0.077$ m away. The answer is (c).

6. For a fixed-fixed string the fundamental wavelength $\lambda_1 = 2L$. If we change the tension, but not the length, the fundamental wavelength will stay the same. The answer is (a).

7. The wavelength of the sound is affected by the motion of the source *only*, not by the motion of the listener. All listeners, whether moving or not, should measure the same wavelength. The frequency measured by a person standing at rest as the car approaches is

   $$f_{\rm L} = \left(\frac{v}{v + v_{\rm S}}\right) f_{\rm S} = \left(\frac{343\ {\rm m/s}}{343\ {\rm m/s} + (-30\ {\rm m/s})}\right) 1.00\ {\rm kHz} = 1.096\ {\rm kHz}$$

   Which corresponds to a wavelength $\lambda = v/f = 0.313$ m. The answer is (d).

8. Intensity decreases as the inverse-square of the distance. The answer is (e).

9. Doubling the distance will decrease the intensity by a factor of four. The change in the sound intensity level $\Delta\beta = (10\ {\rm dB})\log(I_2/I_1) = (10\ {\rm dB})\log(1/4) = -6.0$ dB. The new sound intensity level is $\beta_2 = 80.0\ {\rm dB} - 6.0\ {\rm dB} = 74.0$ dB. The answer is (a).

10. Recalling the electromagnetic spectrum, radio waves have the largest wavelength (lowest frequency), followed by microwaves, infrared, visible, UV, X-rays, and finally gamma rays. The answer is (e).

11. The figure shows total internal reflection, which happens only when $n_2 < n_1$. Having $n_2 = n_1$ is like having no boundary at all. The answer is (b).

12. The ray travelling across the oil is incident at the oil-air interface at an angle

    $$\theta = \tan^{-1}\left(\frac{2.00\ {\rm m} - 0.90\ {\rm m}}{1.88\ {\rm m}}\right) = 30.33^\circ$$

    If this is the critical angle at that interface, then $\theta_c = \sin^{-1}(n_{\rm air}/n_{\rm oil}) \longrightarrow n_{\rm oil} = 1.98$. The answer is (e).

13. For a double-slit $d \sin \theta'_m = (m + 1/2)\lambda$. Increasing d decreases θ, and thus the separation between minima decreases. The answer is (d).

14. Ideally, all the bright fringes of a double-slit interference pattern are equally intense. The answer is (a).

15. 450 lines/mm means a grating spacing $d = 2.22 \times 10^{-6}$ m. The most deviated colour is the one that corresponds to the longest wavelength. We can find the order at which red (700 nm) deviates at 90°: $m = d \sin 90°/\lambda = 3.17$. Rounding down, $m = 3$ is the highest complete order. The answer is (b).

16. For a circular aperture $\theta_1 \approx 1.22\lambda/D$. Doubling the size of the aperture will halve the value of θ_1, but the means the resolving power *doubles* (the smaller θ_1, the greater the resolving power). The answer is (c).

17. The number of photons $N = E_{\text{total}}/E_{\text{photon}} = E_{\text{total}}\lambda/hc = 2.8 \times 10^{18}$ photons. The time is not needed. The answer is (e).

18. Increasing the intensity increases the number of photons hitting the cathode, but has no effect on the energy carried by each photon. So more electrons are released from the cathode, but with the same maximum energy. The answer is (b).

19. According to Wien's law $\lambda_{\text{peak}} = 2.90 \times 10^6\ \text{nm K}/T$. Doubling the temperature halves the peak wavelength. The answer is (b).

20. Using the hydrogen energy levels

$$\frac{hc}{\lambda} = \frac{-13.6\ \text{eV}}{n^2} - \frac{-13.6\ \text{eV}}{2^2} \longrightarrow n = 5$$

The answer is (c).